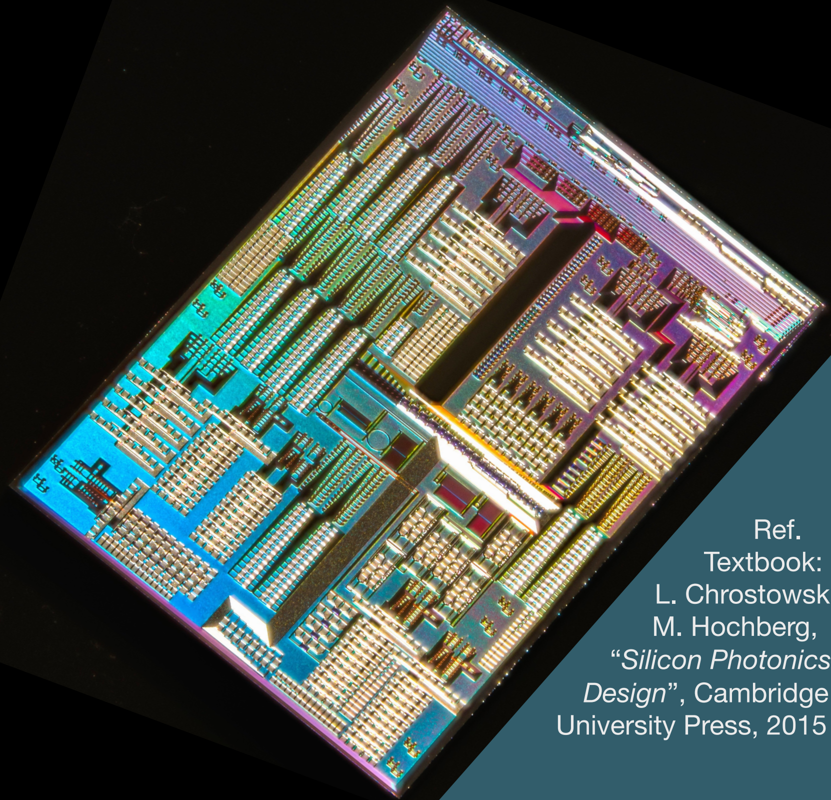
Bragg Reflectors, VCSELs, Transfer Matrix Method, Waveguide Bragg Gratings, Bragg cavity design



Dr. Lukas Chrostowski

Ref. Textbook: L. Chrostowski, M. Hochberg, "*Silicon Photonics Design*", Cambridge University Press, 2015

Week 3

- DURING CLASS:
  - Q&A
  - Fabry-Perot Cavities
  - Bragg gratings and Transfer Matrix Method
- HOMEWORK:
  - Complete “Semiconductor Laser Introduction” > “Fabry Perot Cavities”
  - Complete “Semiconductor Laser Introduction” > “VCSEL Design, Transfer Matrix Method”
  - Complete “Project 1 – Photonic Circuits: Bragg grating cavity Design | Bragg Gratings”

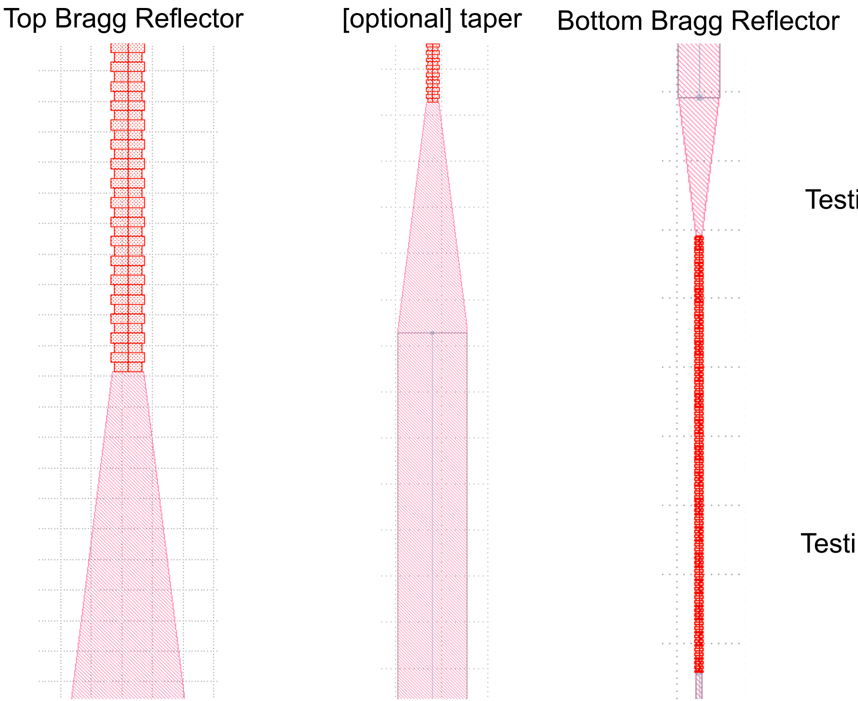
Objective

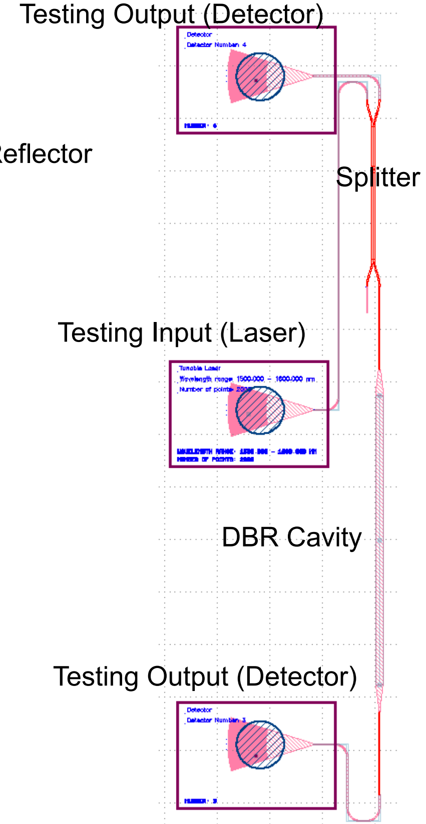
- Understand and model
  - A Fabry-Perot cavity based on two “regular” mirrors
    - HW on edX
      “Semiconductor Laser Introduction | Fabry Perot cavities”
  - Waveguides
    - HW on edX
      “Photonic Components | Waveguides & Waveguide modelling”
  - Bragg gratings
    - HW on edX
      “Photonic Components | VCSEL Design, Transfer Matrix Method”
  - A mirror based on Waveguide Bragg gratings
    - HW on edX
      “Photonic Circuits: Bragg grating cavity Design | Bragg Gratings”
- Course project: your design, gets fabricated and measured; you analyze.
  - A Fabry-Perot cavity with two Bragg gratings and waveguides, operating at 1.31 μm
    - Project report
    - Project design layout - draft, final

Project Report

- Keep your report as concise as possible (e.g., 3-4 pages)
- Here is the template I recommend you use:
  - http://www.ieee.org/conferences_events/conferences/publishing/templates.html
  - Example reports and code to help you: https://www.dropbox.com/sh/r8zvccc7v04qfco/AAB-GZw5rKVmMBC4Ji983KeZa?dl=1
- Please include:
  - Project title, your name
  - Abstract / Introduction - state the objectives of your project
  - Design: what determined your choice of parameters;
  - Images: the important portions of your layout
  - Table: list the parameters for your design; include simulated performance parameters, including FSR, bandwidth, Q, etc.
  - Model: expected results, graphs. You can overlay four plots on a single graph to make the report more concise.
  - Experiments: prior to doing the actual experiments, describe how the devices will be measured, how you will use the de-embedding structures, etc.
  - Appendix: Matlab or Python code you wrote

Our Course Project





Outline

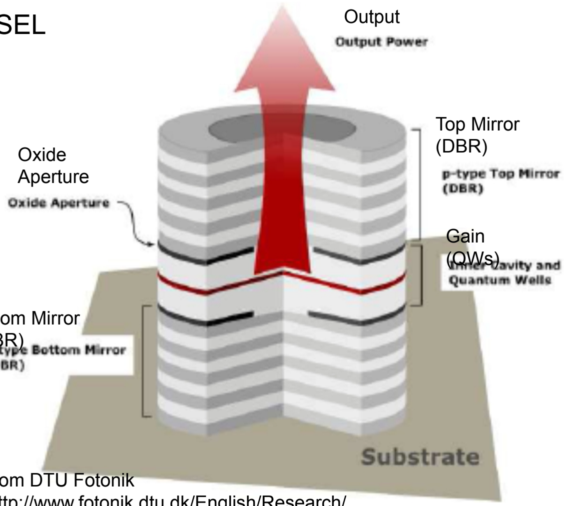
- Introduction to Bragg gratings
- VCSELs
- Modelling VCSELs using the Transfer Matrix Method
  - Matlab implementation; Python version also available (on ELEC413 GitHub: Bragg_TMM).
- Waveguide Bragg grating – unit cell simulations
  - Lumerical FDTD script (on ELEC413 GitHub: Bragg_Bandstructure).
- Waveguide Bragg Grating Cavity Design
  - Transfer Matrix Method approach, using
    - 1) Waveguide model
    - 2) Bragg unit cell model

What are Bragg gratings?

- Excellent optical filters
  - can be designed for many different shapes;
    - narrow vs. broadband
  - wide control of spectral shape
    - thanks to choices in Δn, period, # periods (N)
- Numerous applications
  - lasers – mirrors
    - N = 3-30 for VCSELs
    - N = 100 - 1000s for DFB or DBR lasers
  - filters for communications – in fibres
  - sensors

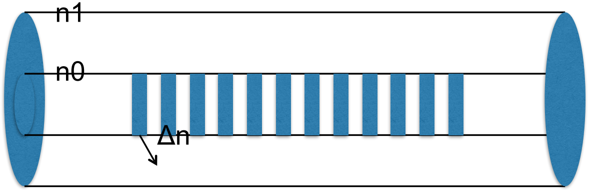
Bragg grating examples

VCSEL

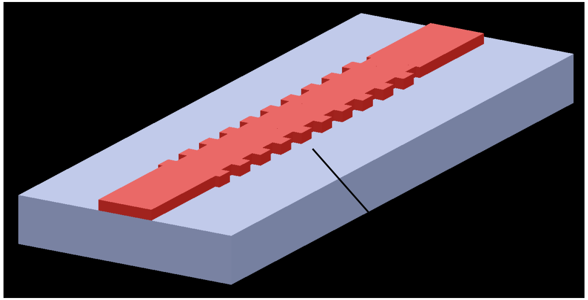


from DTU Fotonik
http://www.fotonik.dtu.dk/English/Research/ResearchActivities/NanoDevices_research/VCSELs.aspx

Fiber Bragg Grating



Waveguide Bragg Grating



Silicon Laser – used by Intel

- **Bragg grating:**
  - Front mirror & back mirror
- **Waveguide**
  - Rib waveguide
- **Gain inside the Cavity**
  - with semiconductor for optical gain
  - electrical contacts

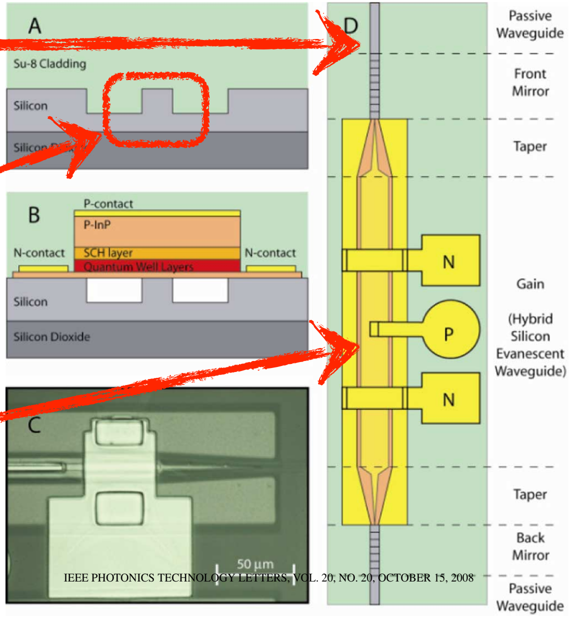
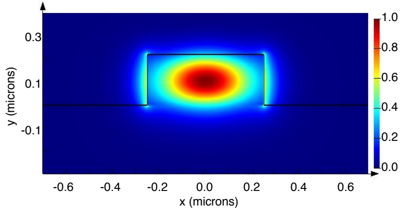


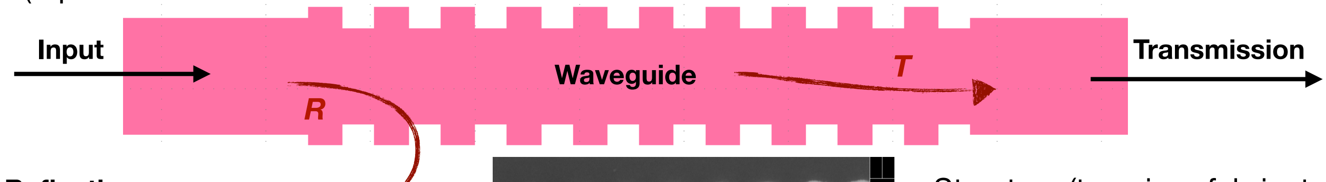
Fig. 1. (a) Passive silicon rib and (b) hybrid silicon evanescent waveguide cross section. (c) Microscope image of a hybrid to passive taper. (d) DBR-SEL top-view topographical structure.

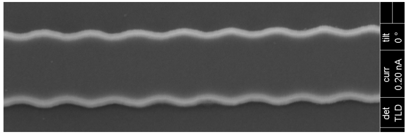
Waveguide Bragg grating

- Structure (side view):



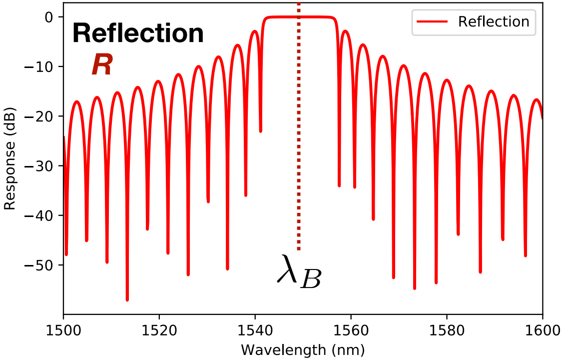
- Structure (top view):

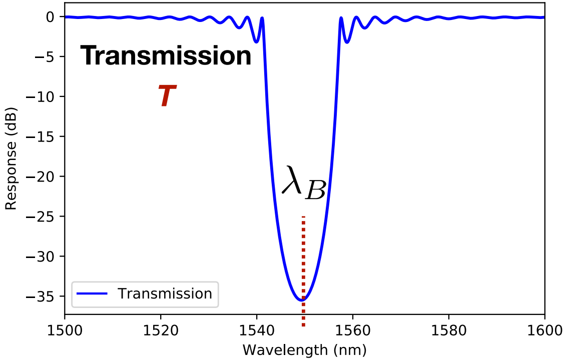




- Structure (top view, fabricated)

- Performance





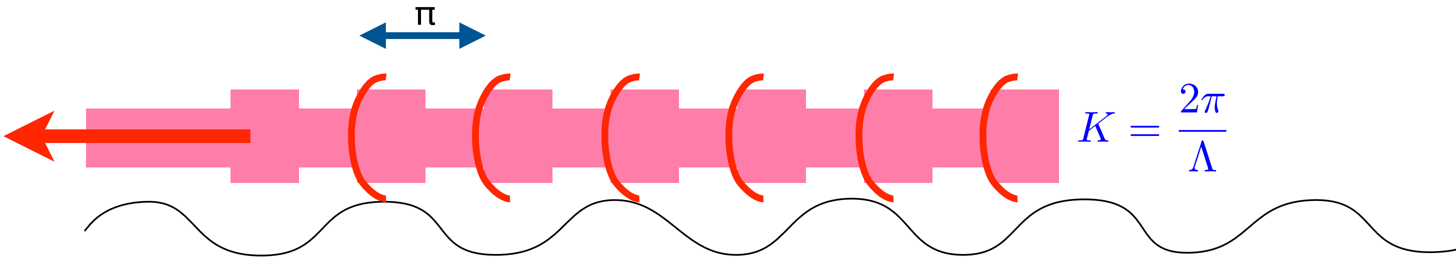
Waveguide Bragg grating – operating wavelength

- Phase matching condition:

$$\beta \cdot 2\Lambda = 2\pi \cdot M$$

- M is the grating order

- Propagation constant X grating period is equal to a 360° (or multiple) phase shift
  - Optical wavelength inside the grating matches 2X period
- Namely, constructive interference from each period, where light has to travel 2 * Period



$$K = \frac{2\pi}{\Lambda}$$

$$\beta = n_{\text{eff}} k_0 = \frac{2\pi n_{\text{eff}}}{\lambda_0}$$

Waveguide Bragg grating – operating wavelength

- Bragg condition – Wave vector matching:

$$K = \frac{2\pi}{\Lambda}$$ Grating, M=1

$$\beta_{\mathrm{left}} = n_{\mathrm{eff}} \cdot k_0 = \frac{2\pi}{\lambda_0} n_{\mathrm{eff}}$$

waveguide propagation constant (backwards)

$$\beta_{\mathrm{right}} = n_{\mathrm{eff}} \cdot k_0 = \frac{2\pi}{\lambda_0} n_{\mathrm{eff}}$$

waveguide propagation constant (forward)

$$\beta_{\mathrm{right}} - K = -\beta_{\mathrm{left}}$$

- We can find the Bragg wavelength:

$$\lambda_B = 2 n_{\mathrm{eff}} \Lambda$$

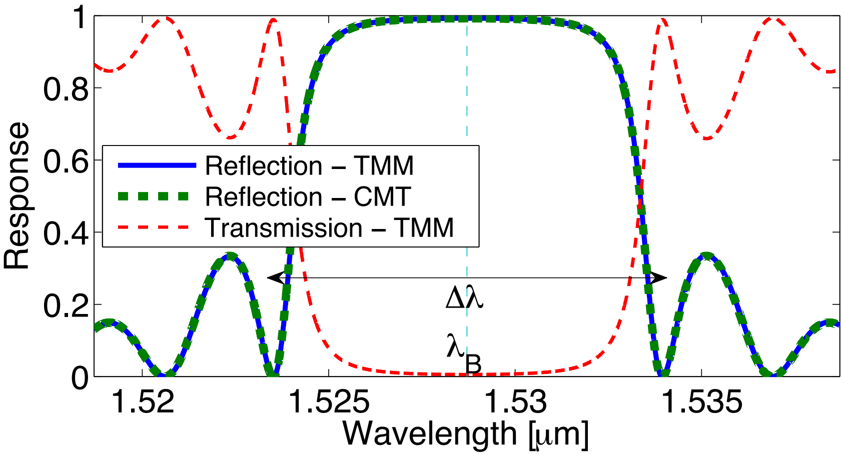
Uniform Bragg grating

- Can have nearly 100% reflectivity over a band
  - R depends on # of gratings, and grating strength (kappa). From Coupled Mode Theory (optional):

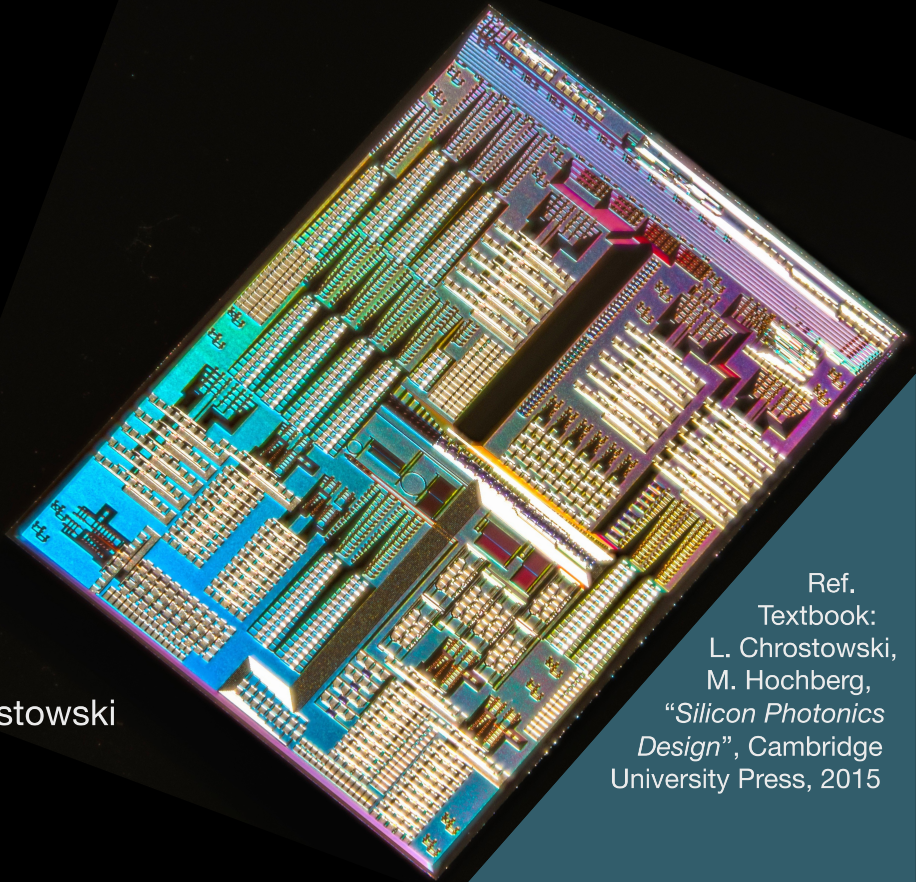
$$R_{peak} = \tanh^2(\kappa L)$$

  - Bandwidth depends mainly on kappa:

$$\Delta\lambda = \frac{\lambda_B^2}{\pi n_g}\sqrt{\kappa^2 + (\pi/L)^2}$$



VCSELs



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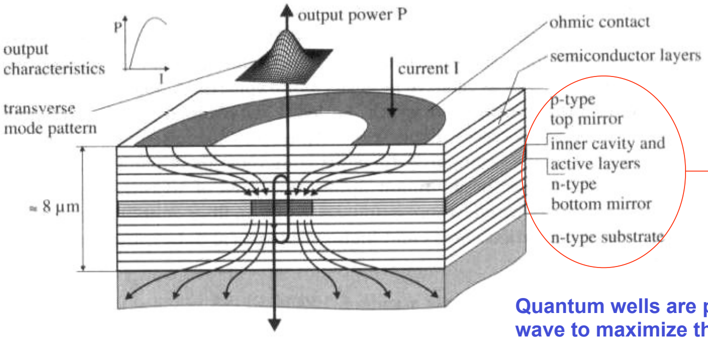
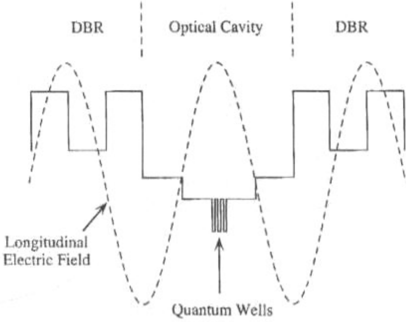
VCSEL Structure

- The motto of design
  - ***"HIGH"*** → high surface reflectivity (>99.9%)
  - ***"SMALL"*** → small active volume
  - ***"WELL CONFINED"***
    → effective optical / electrical confinement
- Schematic layer structure

Source: Bloomberg reporting

Bloomberg



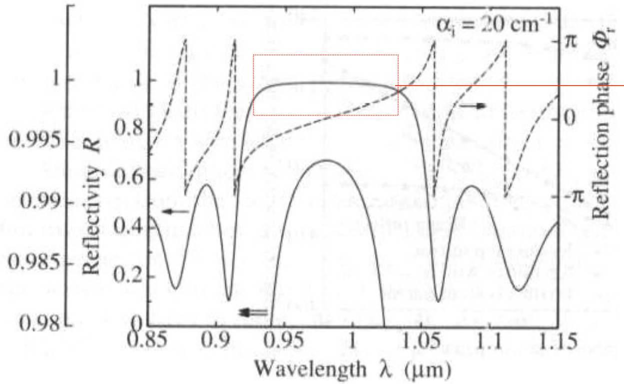


Quantum wells are placed at the antinode of the standing wave to maximize the modal gain

Ref: Michalzik, Ebeling

Bragg Reflectors

- Reflectivity versus wavelength



$$\Delta\lambda_{stop} \approx \frac{2\lambda_B \overline{\Delta n_B}}{\pi < \overline{n_{gr}} >}$$

λ_B Bragg wavelength

$\overline{\Delta n_B} = | n_1 - n_2 |$

$< \overline{n_{gr}} >$ Spatial average of the group index

$(< \overline{n_{gr}} > = \overline{n} - \lambda d\overline{n}/d\lambda)$

- If $\overline{\Delta n_B}$ = 0.56, $< \overline{n_{gr}} >$ = 3.6, and λ_B = 980nm, then $\Delta\lambda_{stop}$ = 100nm

Ref: Michalzik, Ebeling

Bragg Reflectors

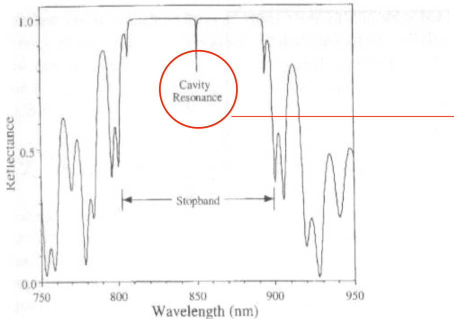
- Longitudinal mode spacing

$$\Delta\lambda_m \approx \frac{\lambda^2}{2L_{eff} < \overline{n_{gr}} >}$$

$$l_{eff} \approx \frac{\lambda_B}{4\Delta \overline{n_B}}$$

Phase penetration depth of the incident wave into the DBRs

$$L_{eff} = L + l_{eff,t} + l_{eff,b}$$



Just a single longitudinal mode can oscillate in a cavity

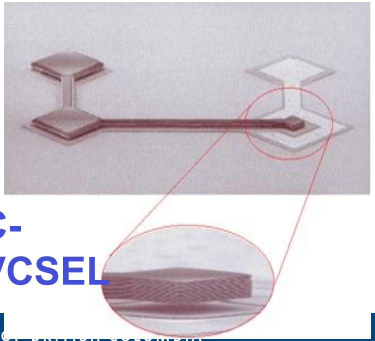
- With $L_{eff} \approx 1.3um$, $\Delta\lambda_m \approx 110nm$, beyond the mirror stop band
- A single mode appears as a sharp dip at reflectivity spectrum that should spectrally overlap the laser gain for min threshold current

Introduction to Tunable VCSEL

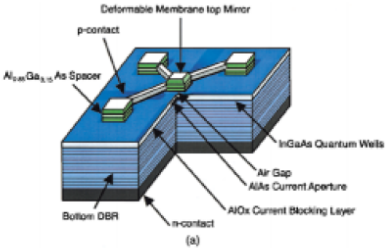
- Methods
  1) By changing refractive index due to temperature or carrier injection
  2) The thickness gradient of layers close to active layer
  3) Micromechanically changing the cavity by applying a reverse bias voltage

  ➔ Lasing Wavelength can be varied by changing cavity length

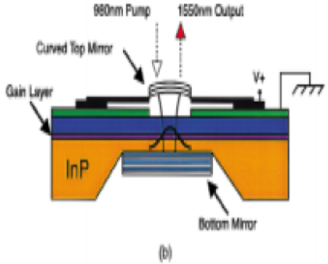
$$<\bar{n}> L = m\lambda / 2$$



C-VCSEL

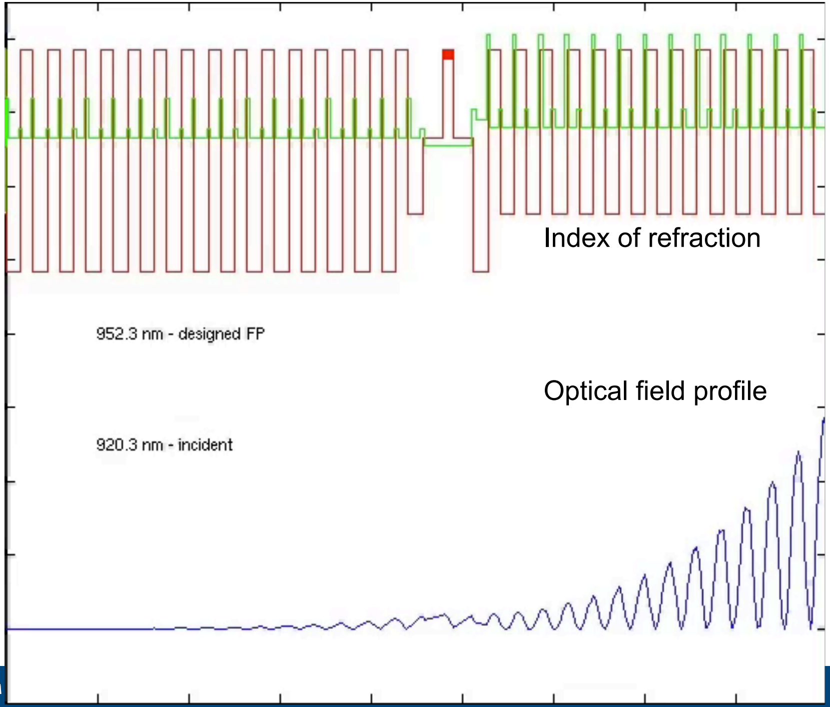


Membrane-VCSEL

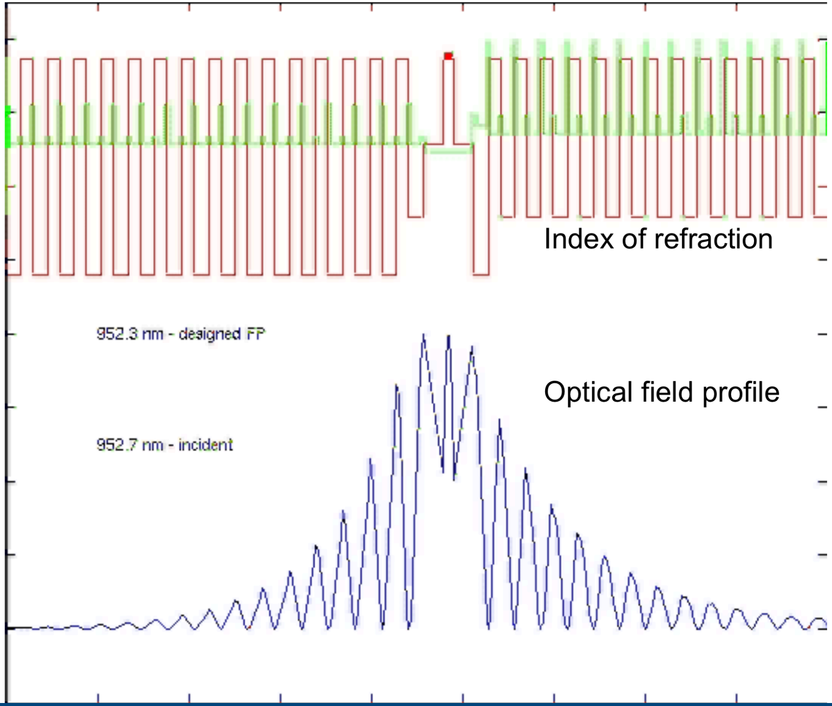


A half-symmetric cavity MEMS-VCSEL

Distributed Bragg Grating Laser (VCSEL)



Distributed Bragg Grating Laser (VCSEL)



VCSEL Homework

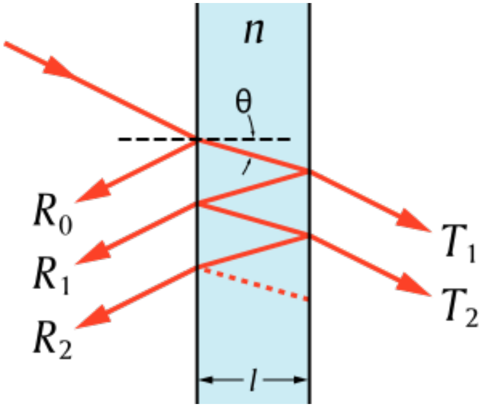
- Design a VCSEL cavity using the **Transfer Matrix Method (TMM)** with the following specs:
  - Single mode operation, with a centre wavelength = 980 nm
- Given:
  - Material loss, 20 cm^{-1}.
  - $\Delta n = 0.56$, $n_{average,group} = 3.6$
- Parameters
  - $R_{bottom} = 99.9\%$
    - Number of DBR layers?
  - R_{top} = start with 99% and adjust
    - Number of DBR layers?
  - Layer thickness values: L_{high}, L_{low}.
    - Note that the Matlab code provided assumes that $L_{high} = L_{low}$. This is not optimal for high-contrast gratings, but ok for weak ones
- edX problem: Due: next Tuesday 12:30.

VCSEL Homework

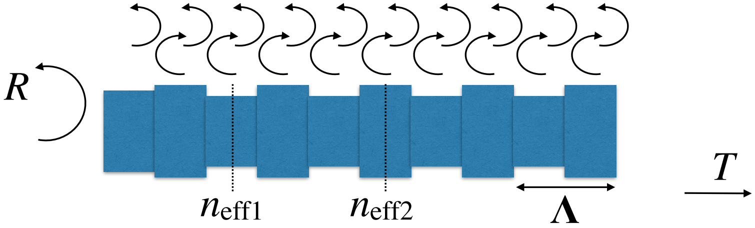
- Steps:
  - Design bottom mirror with R = 99.9%.
    - Bragg = 980 nm; L1=Bragg/4/n1; L2=Bragg/4/n2;
    - Adjust NG (# of periods) so to get to target R
  - Build VCSEL model:
    - Start with FP, and add Tcavity = HomoWG_Matrix (20 cm-1, ?) * HomoWG_Matrix (-1000 cm-1, 50 nm) * HomoWG_Matrix (20 cm-1, ?)
  - Plot spectrum:
    - With gain = 0, should see an FP mode
    - Turn on gain, FP mode should change
    - Adjust R_top (N_top) so that you get a transmission spectrum that is ~1 at the FP mode (i.e., the dip is gone); Gain = Loss

Transfer Matrix Method

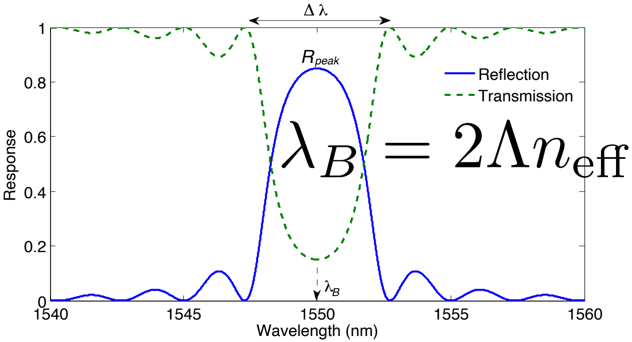
- Read about, including derivation:
  - https://en.wikipedia.org/wiki/Transfer-matrix_method_(optics)
  - Yariv's textbook, and Chrostowski's book.
- Useful technique for solving multi-layer film transmission, e.g.,
  - VCSELs
  - Anti-reflection coating on lenses
- Derived for plane waves
- Can use it for complex filter design
  - including waveguides
  - requires extension of this model



Transfer Matrix Method – Bragg grating



- Consider each reflection one by one
- Construct a matrix representation for all reflections and transmissions

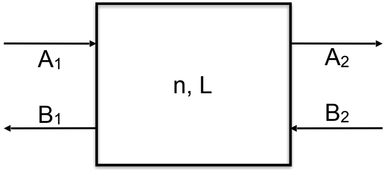


$$\lambda_B = 2\Lambda n_{\mathrm{eff}}$$

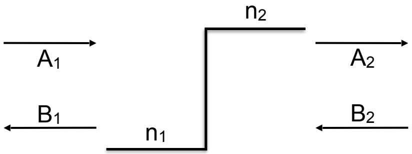
Transfer Matrix Method – Bragg grating

- Matrices:

$$\begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} A_2 \\ B_2 \end{bmatrix}$$



(a) Propagation matrix.



(b) Index step matrix.

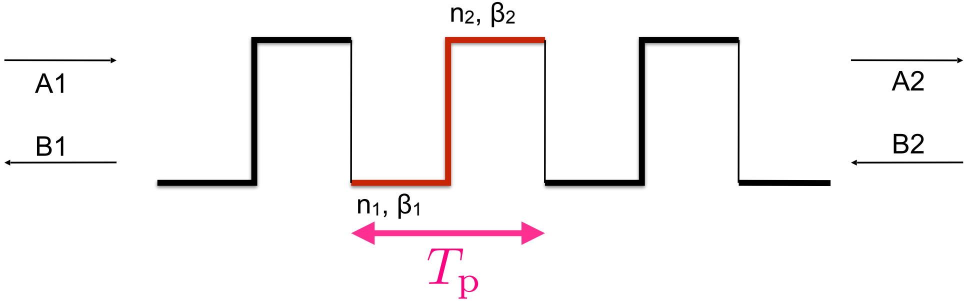
$$T_{\text{hw}} = \begin{bmatrix} e^{j\beta L} & 0 \\ 0 & e^{-j\beta L} \end{bmatrix}$$

$$T_{\text{is-12}} = \begin{bmatrix} 1/t & r/t \\ r/t & 1/t \end{bmatrix} = \begin{bmatrix} \frac{n_1+n_2}{2\sqrt{(n_1 n_2)}} & \frac{n_1-n_2}{2\sqrt{(n_1 n_2)}} \\ \frac{n_1-n_2}{2\sqrt{(n_1 n_2)}} & \frac{n_1+n_2}{2\sqrt{(n_1 n_2)}} \end{bmatrix}$$

$$\beta = \frac{2\pi n_{\text{eff}}}{\lambda} - i\frac{\alpha}{2}$$

Transfer Matrix Method – Bragg grating

- Uniform Periodic structure, one period:



- Uniform Periodic structure, NG periods: $T_\mathrm{p} = T_\text{hw-1} T_\text{is-12} T_\text{hw-2} T_\text{is-21}$

$$T_\mathrm{total} = (T_\mathrm{p})^{NG} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \rightarrow \begin{array}{l} T = \left(\dfrac{1}{T_{11}}\right)^2 \\ R = \left(\dfrac{T_{21}}{T_{11}}\right)^2 \end{array}$$

Example

- Using Matlab (also in Python in ELEC413 GitHub: Bragg_TMM)
- Plot the spectrum of R and T

- Parameters

```
function Grating_Parameters
%Set the parameters

global Bragg Period NG L delta_n n1 n2 loss;

Bragg=1550e-9;   % Bragg wavelength
Period=310e-9;   % Bragg period
n_eff=Bragg/(2*Period); % Average effective index

NG=500;      % Number of grating periods
L=NG*Period;     % Grating length

delta_n=0.01;     % Index contrast between n1 and n2
n1=n_eff-delta_n/2;
n2=n_eff+delta_n/2;

loss=0;
```

Example

- Calculate the transfer matrix of a homogeneous section

```
function T_hw=HomoWG_Matrix(wavelength,l,neff,loss)
% Calculate the transfer matrix of a homogeneous waveguide.

Grating_Parameters;

%Complex propagation constant
beta=2*pi*neff/wavelength-1i*loss/2;

v=[exp(1i*beta*l) exp(-1i*beta*l)];
T_hw=diag(v);
```

- Calculate the transfer matrix of a refractive index step

```
function T_is=IndexStep_Matrix(n1,n2)
% Calculate the transfer matrix for a index step from n1 to n2.

a=(n1+n2)/(2*sqrt(n1*n2));
b=(n1-n2)/(2*sqrt(n1*n2));
T_is=[a b; b a];
```

Example

- Calculate the total transfer matrix for a certain wavelength

```
function T=Grating_Matrix(wavelength)
% Calculate the total transfer matrix of the gratings

global Period NG;
global n1 n2 loss;

l=Period/2;
T_hw1=HomoWG_Matrix(wavelength,l,n1,loss);
T_is12=IndexStep_Matrix(n1,n2);
T_hw2=HomoWG_Matrix(wavelength,l,n2,loss);
T_is21=IndexStep_Matrix(n2,n1);
Tp=T_hw1*T_is12*T_hw2*T_is21;
T=Tp^NG;
```

- Calculate the R and T

```
function [R,T]=Grating_RT(wavelength)
%Calculate the R and T for a certain wavelength

M=Grating_Matrix(wavelength);

T=abs(1/M(1,1))^2;
R=abs(M(2,1)/M(1,1))^2;
```

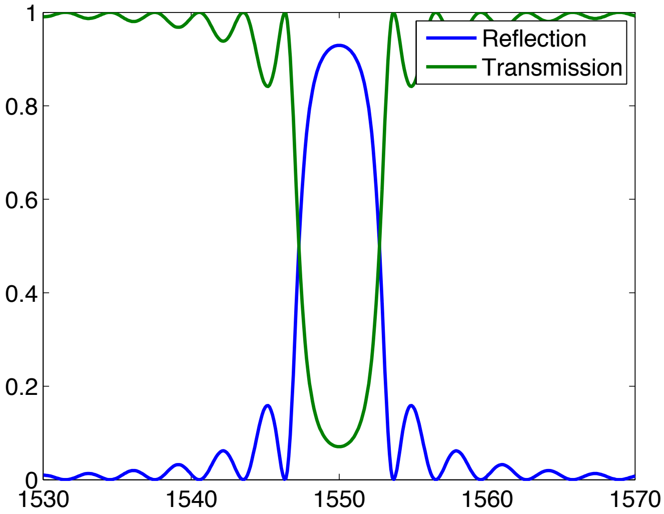
Example

- Main file

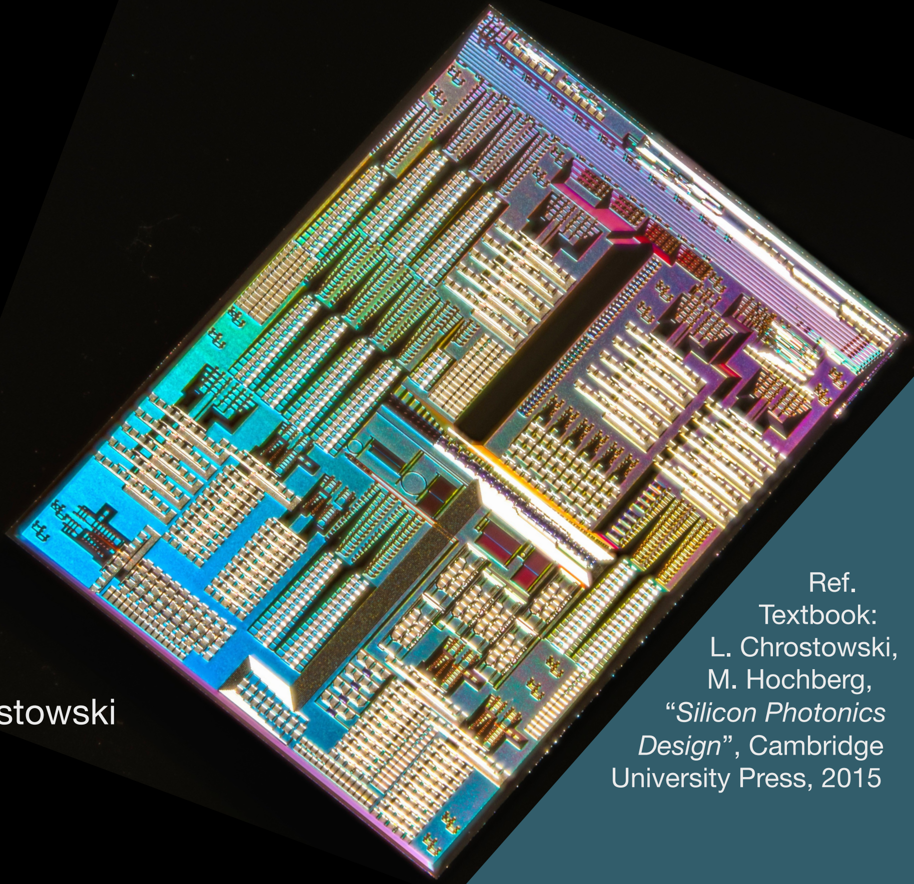
```
function Grating
%This file is used to plot the reflection/transmission spectrum.
clear;
clc;
global Bragg;
Grating_Parameters;

span=40e-9;  % Set the wavelength span for the simultion
resolution=0.1e-9;  % Set the wavelength resolution
N=span/resolution;
Lambda=zeros(N+1,1);
R=zeros(N+1,1);
T=zeros(N+1,1);

for i=1:N+1
    wavelength=Bragg+(i-1-N/2)*resolution;    % Wavelength sweep
    [r,t]=Grating_RT(wavelength); % Calculate the R and T
    Lambda(i)=wavelength*1e9; % in nm
    R(i)=r;
    T(i)=t;
end

figure;
plot(Lambda,[R T],'LineWidth',2);
```

Example

- Results



Waveguide Bragg Gratings



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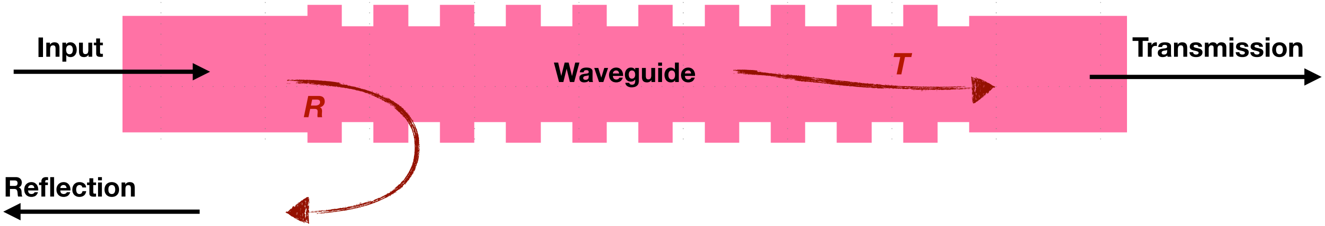
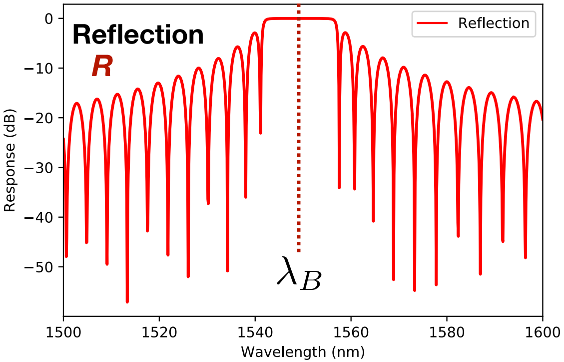
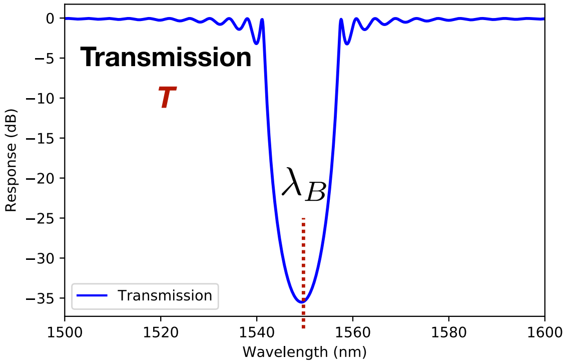
Waveguide Bragg grating

- Structure:



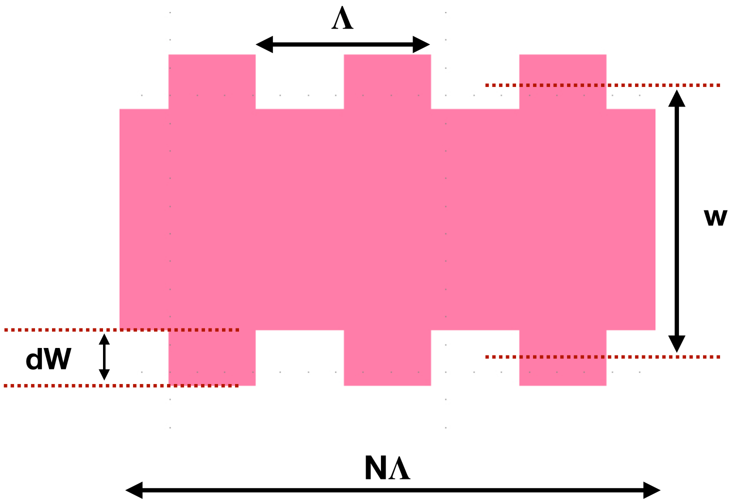
- Performance





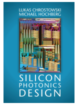
Waveguide Bragg grating – parameters

- Parameters
  - Λ: Grating period
  - w: width of the waveguide
  - dW: corrugation width
  - Type: Rectangular or sinusoidal
  - N: number of grating periods



Waveguide Bragg grating – optical spectrum

Ref: Chrostowski, Hochberg, "Silicon Photonics Design" *Cambridge Press* (2015)



- Calculation of the optical transmission spectrum for a uniform grating, from coupled-mode theory:

$$r = \frac{-i\kappa \sinh(\gamma L)}{\gamma \cosh(\gamma L) + i\Delta\beta \sinh(\gamma L)} \tag{4.29}$$

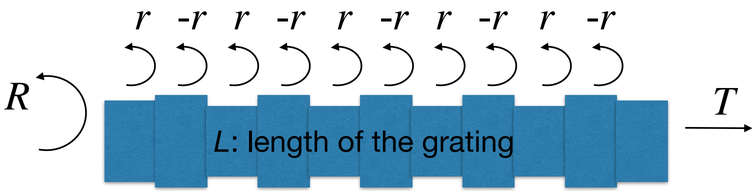
with

$$\gamma^2 = \kappa^2 - \Delta\beta^2 \tag{4.30}$$

Here, $\Delta\beta$ is the propagation constant offset from the Bragg wavelength:

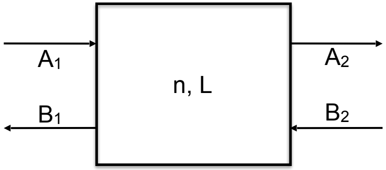
$$\Delta\beta = \beta - \beta_0 << \beta_0 \tag{4.31}$$

and κ is often defined as the coupling coefficient of the grating and can be interpreted as the amount of reflection per unit length.

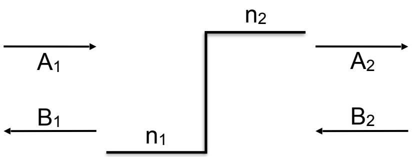


Transfer Matrix Method – Bragg grating

- Matrices:



(a) Propagation matrix.



(b) Index step matrix.

$$T_{\text{hw}} = \begin{bmatrix} e^{j\beta L} & 0 \\ 0 & e^{-j\beta L} \end{bmatrix}$$

$$\beta = \frac{2\pi n_{\text{eff}}}{\lambda} - i\frac{\alpha}{2}$$

$$T_{\text{is-12}} = \begin{bmatrix} 1/t & r/t \\ r/t & 1/t \end{bmatrix} = \begin{bmatrix} \frac{n_1+n_2}{2\sqrt{(n_1 n_2)}} & \frac{n_1-n_2}{2\sqrt{(n_1 n_2)}} \\ \frac{n_1-n_2}{2\sqrt{(n_1 n_2)}} & \frac{n_1+n_2}{2\sqrt{(n_1 n_2)}} \end{bmatrix}$$

Finding r and t using n1-n2 is valid only for plane waves (Fresnel coefficients).

Relate r & t to kappa found from experiments or FDTD

$$\boxed{\kappa = \frac{2r}{\Lambda} = \frac{2}{\Lambda}\frac{\Delta n}{2n_{\text{eff}}}} = \frac{2\Delta n}{\lambda_B}, \quad \Delta n = \kappa \lambda_B / 2$$

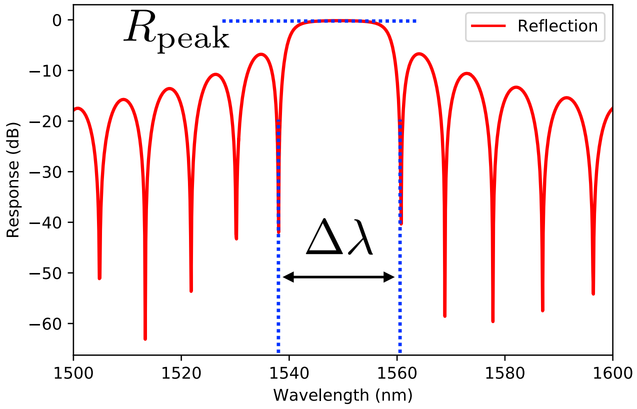
Waveguide Bragg grating – optical spectrum

- Coupled-mode theory predicts the peak reflectivity

$$R_{peak} = \tanh^2(\kappa L)$$

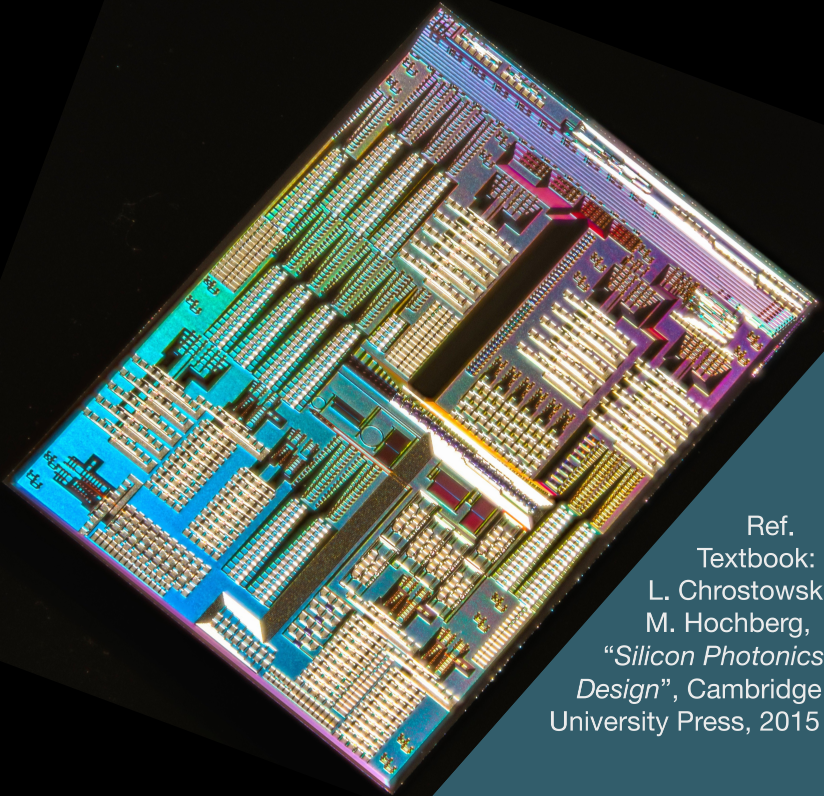
- and the bandwidth (defined here as the 1st-nulls bandwidth, not the 3-dB bandwidth)

$$\Delta\lambda = \frac{\lambda_B^2}{\pi n_g}\sqrt{\kappa^2 + (\pi/L)^2}$$



- **How do we find κ (kappa), the coupling coefficient?**
  - Experiments
  - Simulations

Waveguide Bragg gratings – simulations of a unit cell



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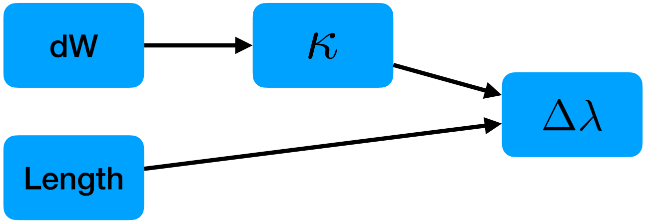
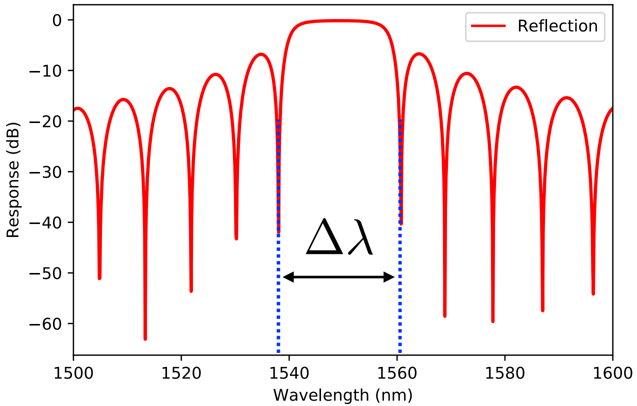
Ref. Textbook: L. Chrostowski, M. Hochberg, “*Silicon Photonics Design*”, Cambridge University Press, 2015

Waveguide Bragg grating – coupling coefficient

- Relationship between physical parameters, model parameters, and performance parameters:



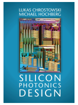
$$\Delta\lambda = \frac{\lambda_B^2}{\pi n_g}\sqrt{\kappa^2 + (\pi/L)^2}$$



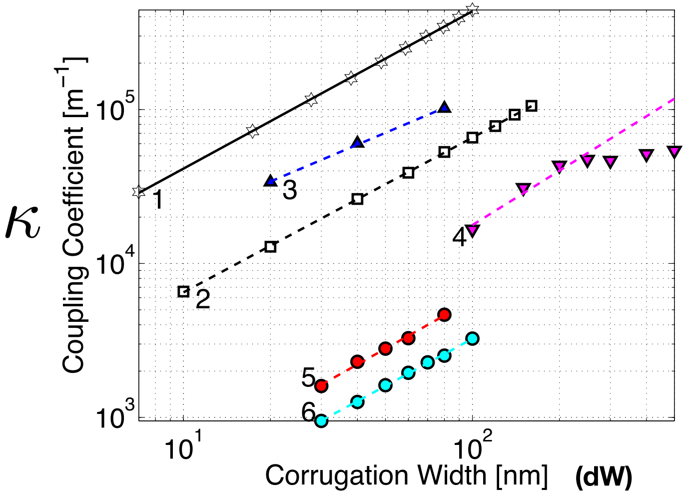
- We need a method of finding the model and performance parameters from the physical parameters
  - Experiments
  - Simulations
    - **Band-structure calculation through 3D-FDTD**
    - CMT-based perturbation analysis
    - Δneff eigenmode approach

Waveguide Bragg grating – Experimental data – κ

Ref: Chrostowski, Hochberg, "Silicon Photonics Design" *Cambridge Press* (2015)



- Experimental data – κ – coupling coefficient:



– 500 x 220 nm waveguide
– oxide cladding

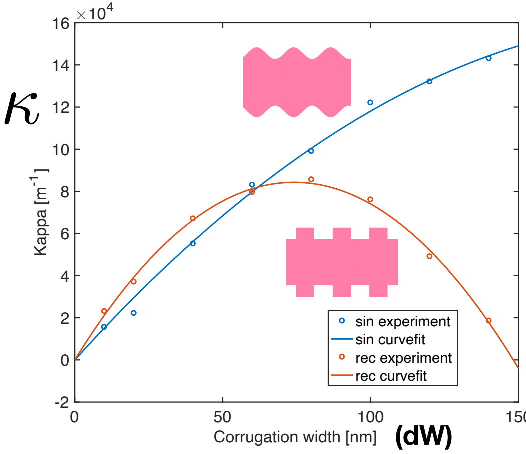
Strip waveguides:
1) TMM + Δn from eigenmode
2) 193 nm litho (imec)
3) EBeam
4) 248 nm litho (IME)

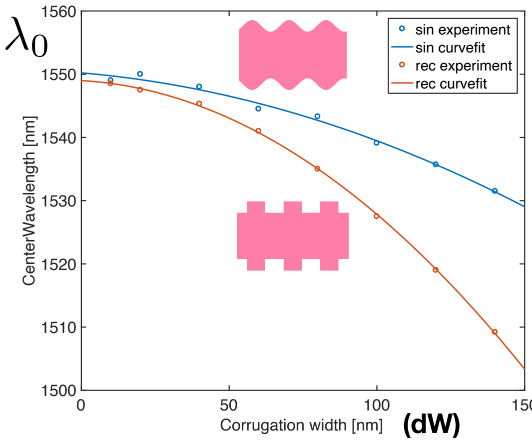
Rib waveguides:
5) rib corrugation (IME)
6) slab corrugation (IME)

- This graph allows you to design a filter with the correct bandwidth for different fabrication processes.

Waveguide Bragg grating – Experimental data – EBeam

- Experimental data for EBeam fabrication, for rectangular and sinusoidal gratings:





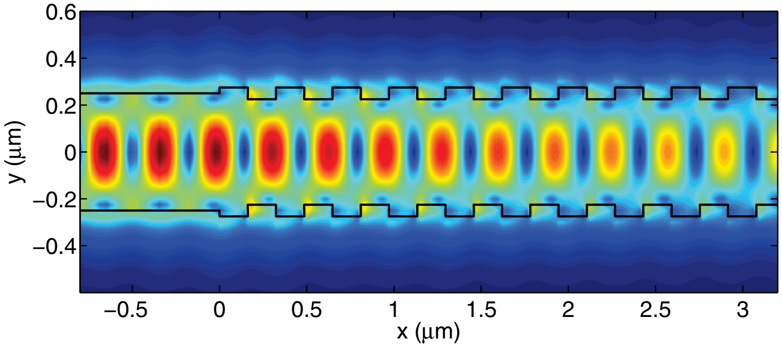
$$\kappa_{\text{sinusoidal}} = -3.7465 \times 10^{18} dW^2 + 1.5555 \times 10^{9} dW$$
$$\kappa_{\text{rectangular}} = -1.53519 \times 10^{19} dW^2 + 2.2751 \times 10^{12} dW$$

$$\lambda_{B,\text{sinusoidal}} = -6.7549 \times 10^{5} dW^2 - 0.0399 dW + 1.5502 \times 10^{-6}$$
$$\lambda_{B,\text{rectangular}} = -1.9 \times 10^{2} dW^2 - 0.0259 dW + 1.549 \times 10^{-6}$$

- Not only does the grating coupling coefficient vary, but so does the central wavelength (“parasitic effect”)
- These compact models are used in the SiEPIC-EBeam-PDK on GitHub

Waveguide Bragg grating – Simulations – 3D FDTD

- We can perform 3D FDTD simulations of the grating

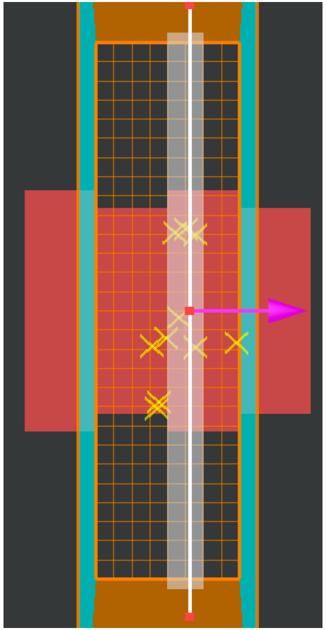


- But an accurate simulation for a full grating, e.g. 100 µm long, takes a long time (hours)
  - Another approach: exploit periodicity to only simulate one period

Waveguide Bragg grating – Simulations – 3D FDTD unit cell

- We can perform 3D FDTD simulations of a unit cell (one period) to find the grating coupling coefficient
  - Periodic structure with a unit cell, infinite length
- Simulation is fast (minutes)
- Can perform parameter sweeps (width, Δw, period)
- Extract centre wavelength and grating strength (kappa, Δn_{eff}, r, bandwidth)

- Subsequently, use the Transfer Matrix Method to find the spectrum for a finite-length Bragg grating



Ref: X. Wang, et al., (Lumerical)
Optics Letters, **39**, 19 (2014)

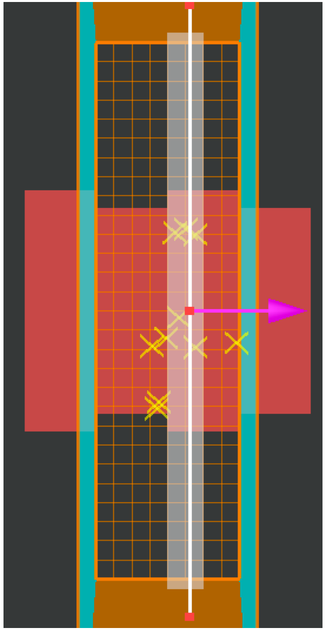
Waveguide Bragg grating – Simulations – 3D FDTD unit cell

- Simulation steps:
  - Draw the structure
  - Define a unit cell
  - Bloch boundary conditions: simulates an infinitely-long grating
  - Set k (wave vector)
  - Excitation source
  - Use time-domain monitors and calculate the optical spectrum
  - Find peaks in the spectrum: these correspond to the 1st-null bandwidth
  - Find Kappa from the bandwidth

$$\Delta\lambda = \frac{\lambda_B^2}{\pi n_g}\sqrt{\kappa^2 + (\pi/L)^2}$$

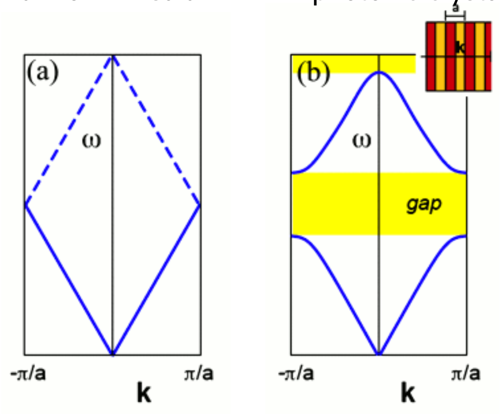
    - and where L is infinity
  - The grating coupling coefficient is:

$$\kappa = \pi n_g \frac{\Delta\lambda}{\lambda_B^2}$$



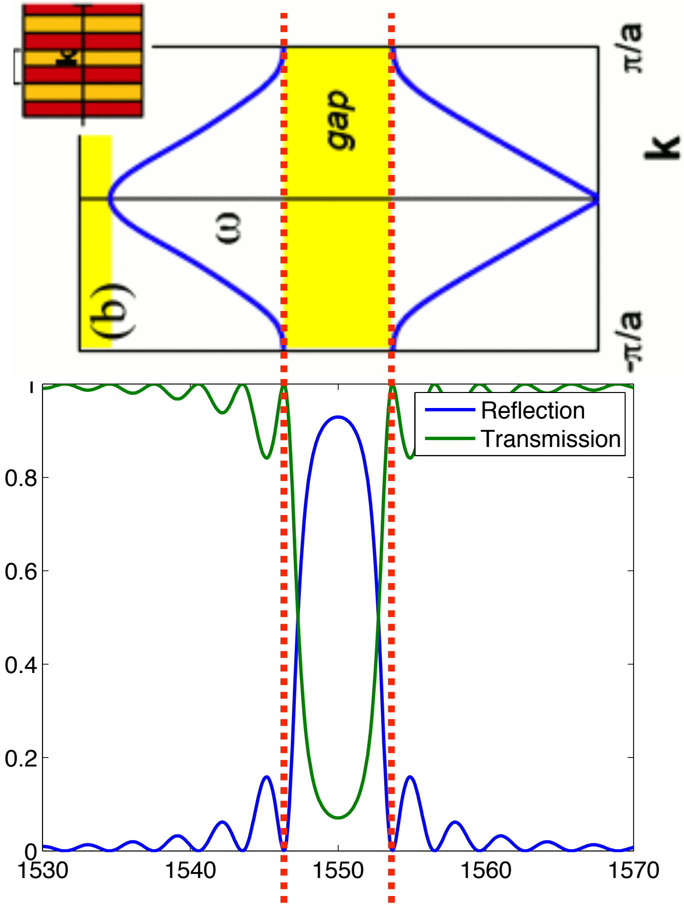
Ref: X. Wang, et al., (Lumerical)
Optics Letters, **39**, 19 (2014)

uniform medium 1D photonic crystal



- Photonic crystals devices have band gaps in which there are no propagating solutions
- The size and location of the gap will give us the center wavelength and bandwidth of the Bragg grating

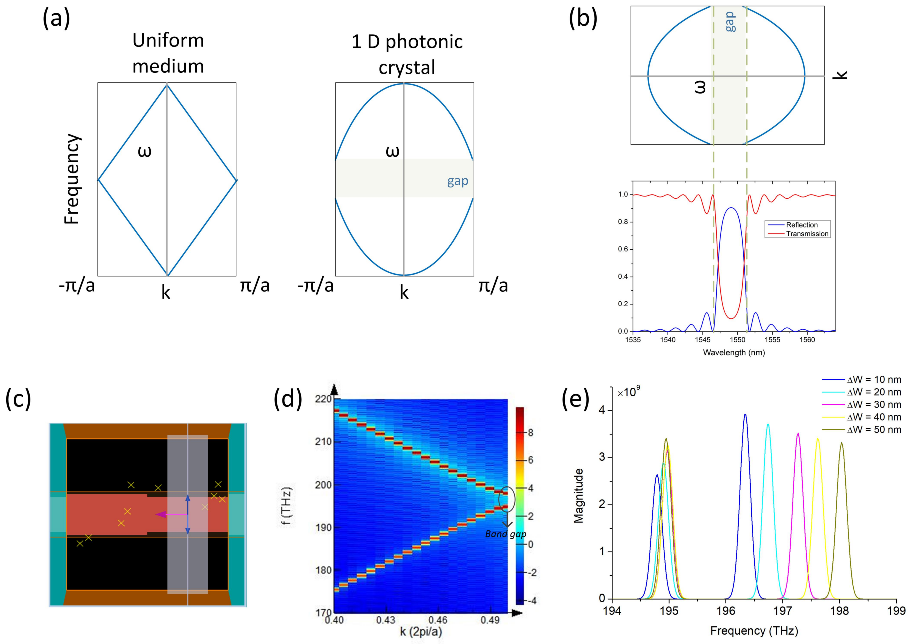
J. D. Joannopoulos *et al*, *Molding the Flow of Light* (Princeton University Press, 1995).

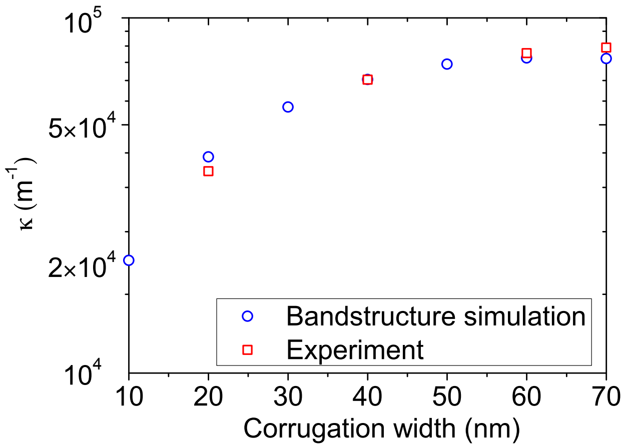


(b)
gap
ω
k
π/a
-π/a
Reflection
Transmission
0
0.2
0.4
0.6
0.8
1
1530
1540
1550
1560
1570

Finding the relationship between ΔW and coupling coefficient (κ)

- Band-structure calculation through 3D-FDTD



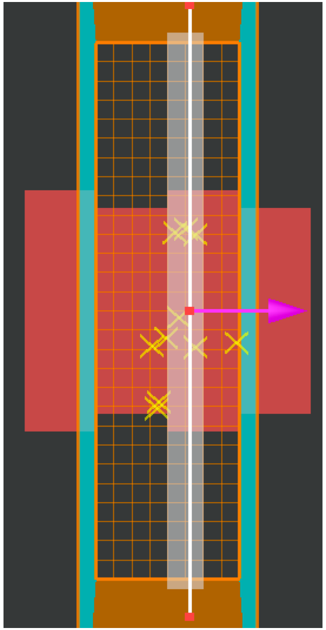


Experiment = 3D FDTD Band-structure

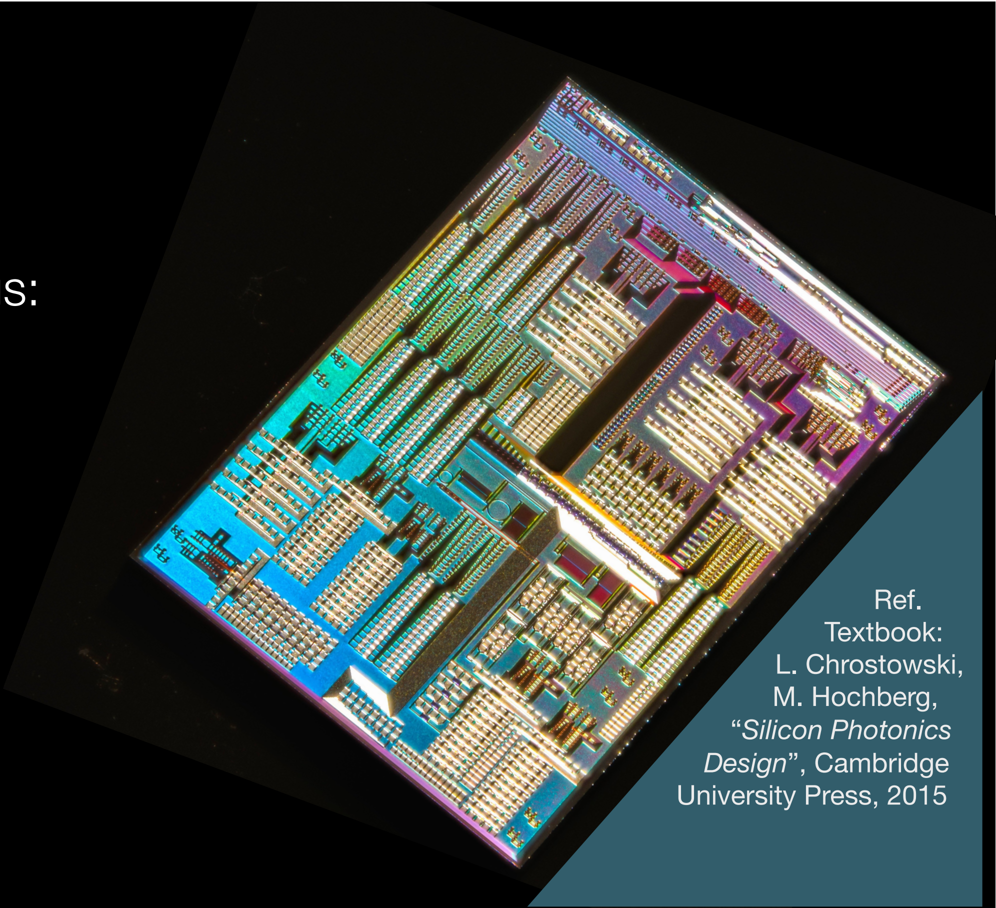
S. Kaushal, et al. Frontiers of Optoelectronics 11.2 (2018): 163-188.
Rui Cheng, PhD Thesis (2020)

Waveguide Bragg grating – Simulations – 3D FDTD unit cell

- Simulation project in UBC-ELEC413 GitHub, Simulations/Bragg_Bandstructure/script_base
  - Open Lumerical FDTD
  - Drag folder into Lumerical FDTD window
  - Edit MAIN_bandstructure.lsf:
    - wavelength range, wl_min, wl_max (configure this for 1.31 μm)
    - mesh: accuracy (1 is for debugging, 4 is very accurate, 2 is ok)
    - ng: group index of the waveguide (average width)
    - W: waveguide width
    - dW: waveguide corrugation



Waveguide Bragg gratings: Simulations using the Transfer Matrix Method



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Ref. Textbook: L. Chrostowski, M. Hochberg, "*Silicon Photonics Design*", Cambridge University Press, 2015

Design Flow

1) Matlab TMM (simplest model): get desired spectrum
   [$n_{eff\text{-}average}$, Δn]

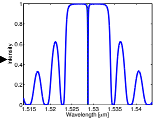
Index matrix

$$\begin{bmatrix} n_1 & L_1 \\ \vdots & \vdots \\ n_i & L_i \\ \vdots & \vdots \\ n_N & L_N \end{bmatrix}$$

1

• TMM

TMM Prediction



3) Matlab TMM: update based on Bragg waveguide parameters
4) Develop a design

2a

• MODE Calculations

2b

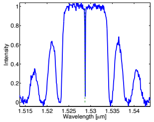
2) Based on physical parameters Δw, w:
   a) MODE: waveguide model with wavelength dependance, and
   b) FDTD and/or Experiments for [κ, Δn_{eff}]
   c) estimated waveguide loss

$$\begin{bmatrix} w_{a,1} & w_{b,1} & L_1 \\ \vdots & \vdots & \\ w_{a,i} & w_{b,i} & L_i \\ \vdots & \vdots & \\ w_{a,N} & w_{b,N} & L_N \end{bmatrix}$$

Physical structure matrix

Mask layout

Experiments



5) Mask layout
6) Verify using INTERCONNECT

Transfer Matrix Method – Bragg grating

- 1) Use the definition of coupling coefficient (= reflections per unit length), and the normal incidence Fresnel reflection coefficient, to find an equivalent Δn:

$$\kappa = \frac{2r}{\Lambda} = \frac{2}{\Lambda}\frac{\Delta n}{2n_{\mathrm{eff}}} = \frac{2\Delta n}{\lambda_B}, \quad \Delta n = \kappa\lambda_B/2$$

  - Use this Δn value in TMM

- 2) Use a wavelength-dependant waveguide model for the effective index, n_{eff}:

$$n_{\mathrm{eff}} = n_1 + n_2\left(\lambda - \lambda_0\right) + n_3\left(\lambda - \lambda_0\right)^2$$

  - e.g. strip waveguide parameters (do this for 1.31 µm wavelength):

$$\lambda_0 = 1.55, n_1 = 2.4445, n_2 = -1.12733, n_3 = -0.033342$$

  - Waveguide dispersion has a big impact on the spectrum of the waveguide Bragg grating

- Construct arbitrary non-uniform structures: Fabry-Perot cavities, etc.

neff - wavelength dependant model

```
function Grating_Parameters
%Set the parameters
global Bragg Period NG L delta_n n1 n2 loss neff;
global wavelength;

Bragg=1550e-9;  % Bragg wavelength
Period=317e-9;  % Bragg period
n_eff=Bragg/(2*Period); % Average effective index

lambda = wavelength*1e6;
lambda0 = 1.55; n1=2.44509955913786;
n2=-1.127332509182173; n3=-0.033342989811319;
% these are constants from the waveguide model.
n_eff = n1 + n2*(lambda-lambda0) + n3*(lambda-lambda0)^2;
```

What value of n_{eff} is required to achieve the desired Bragg wavelength?

Realistic waveguide model. Bragg wavelength unknown and needs spectral simulation.

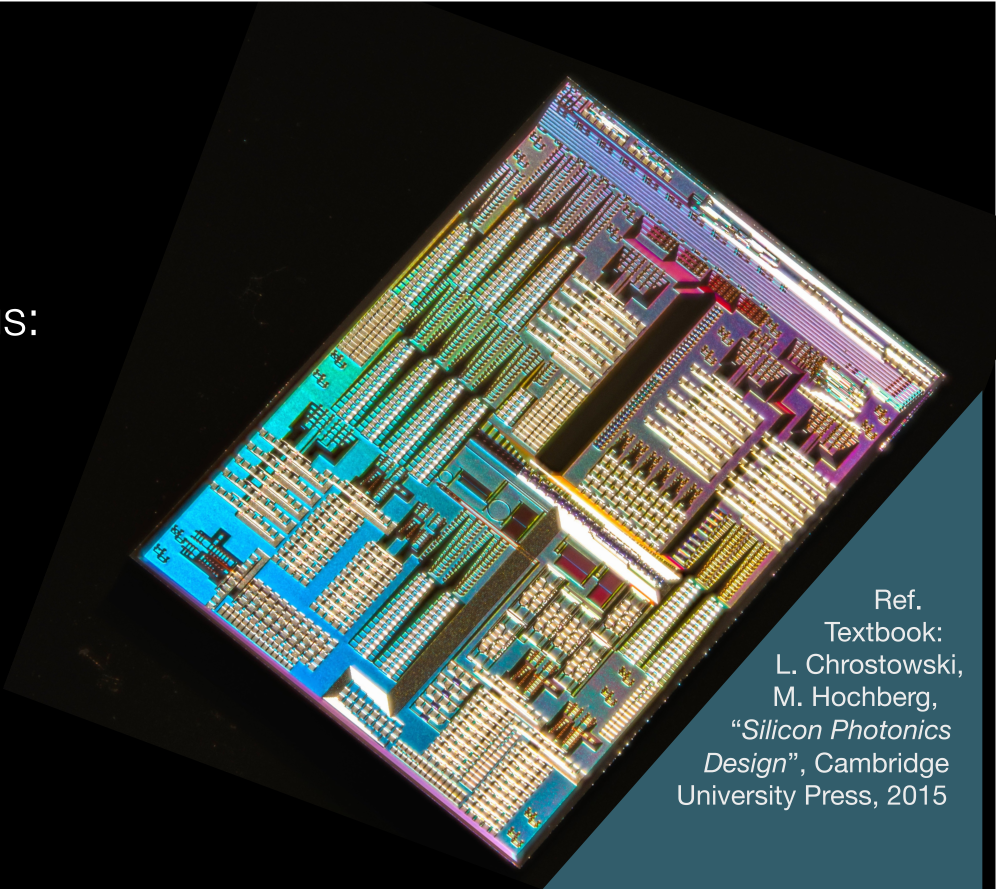
Homework on edX

- Module: Project 1 – Photonic Circuits: Bragg Grating cavity > Bragg Gratings > Transfer Matrix Method for Bragg Gratings

- First two questions
  - Constant index of refraction
- Last two questions
  - n_{eff} wavelength dependant

Waveguide Bragg gratings: Designing Cavities

Dr. Lukas Chrostowski

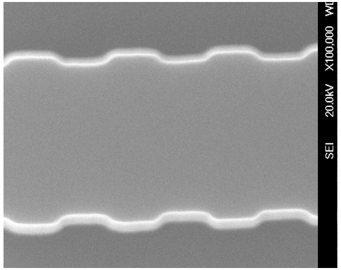


Ref. Textbook: L. Chrostowski, M. Hochberg, "*Silicon Photonics Design*", Cambridge University Press, 2015

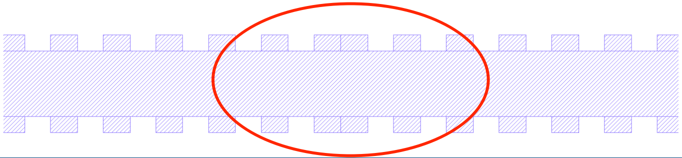
Waveguide Bragg Gratings – Xu Wang

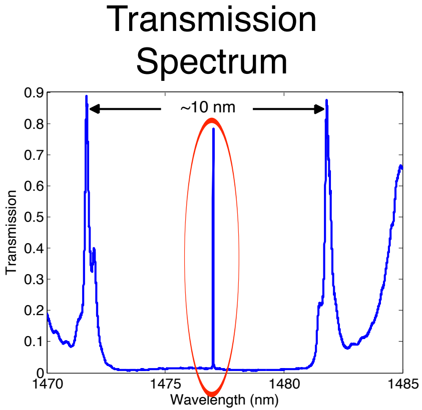
PhD Thesis: https://circle.ubc.ca/handle/2429/45687

- Uniform Bragg gratings
  - makes a mirror (with a bandwidth, typically 1 to 30 nm)



- Phase shifted gratings
  - results in a cavity





Maximum theoretically possible Q

- Quality factor definition:

$$Q = \omega \cdot \tau_p, \qquad \tau_p^{-1} = \alpha \frac{c}{n_g}$$

where ω is the angular frequency, and α is the total power loss in *m^{-1}* including **propagation loss and mirror loss**.

  - What if you had no mirror loss? What would R be?

- Thus,

$$Q = 2\pi \frac{c}{\lambda} \frac{n_g}{c} \frac{1}{\alpha} = 2\pi \frac{n_g}{\lambda \cdot \alpha}$$

- This is the Q given the total “distributed” optical losses

Maximum theoretically possible Q

- Assuming a propagation loss of 3 dB/cm (and no mirror loss), we can find α is the total power loss in *m*^-1^

$$loss_{dB/cm} = -20log_{10}\left(e^{-loss_{m^{-1}}*0.01m}\right)$$

$$\alpha = \ln(10)\frac{3}{10}100 = 69m^{-1}$$

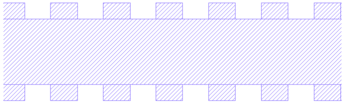
- This allows us to find the maximum theoretical Q factor that can be achieved:

$$Q = 2\pi\frac{4.2}{1.55\times10^{-6}\cdot69} = 247,000$$

This is the situation where the Bragg mirrors are infinitely long.
But we need a finite Bragg grating in order to have light go in/out, and be able to measure it, so the real Q will be lower.

Example Designs – for discussion…

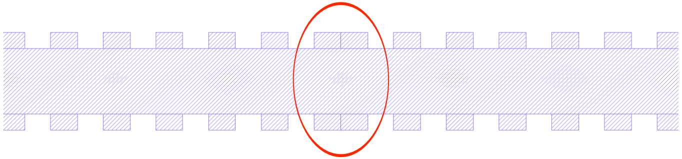
- Case 1:
  - Bragg grating with
    - Δw = 80 nm
    - Number of gratings = 1000
    - Period = 320 nm
  - Waveguide width = 500 nm



- Questions:
  - Sketch R vs. wavelength
  - Peak reflectivity?
  - Centre wavelength?

Example Designs – for discussion…

- Case 2:
  - Bragg gratings with
    - Δw = 80 nm
    - Number of gratings = 1000
    - Period = 320 nm
  - Waveguide width = 500 nm
  - **Fabry-Perot cavity consisting of two Bragg gratings, with a cavity length of 320 nm.**

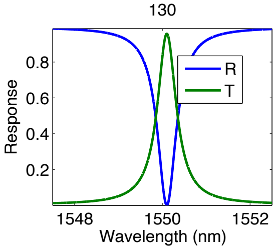


- Questions:
  - Sketch T vs. wavelength
  - Resonator wavelength?
  - Quality factor?

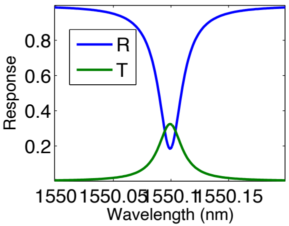
Problem with design?

Insertion Loss considerations

- Is the peak well resolved, with a high amplitude?



- Or is the transmission very low and the Q is not defined (for transmission spectrum, T)?



Matlab simulations

- $\Delta n = 0.04$
- Loss = 3 dB/cm
  - `loss=log(10)*3/10*100;`
- Phase-shifted cavity (extra π/2 shift)
  - `T=Tp^NG * (T_hw2)^1 * Tp^NG * T_hw2;  % insert a high index region.`
- Note: the following simulations did not take into account the wavelength dependance of the effective index (i.e., group index = effective index approximation). So the Q value is incorrect by a factor of ng/neff.

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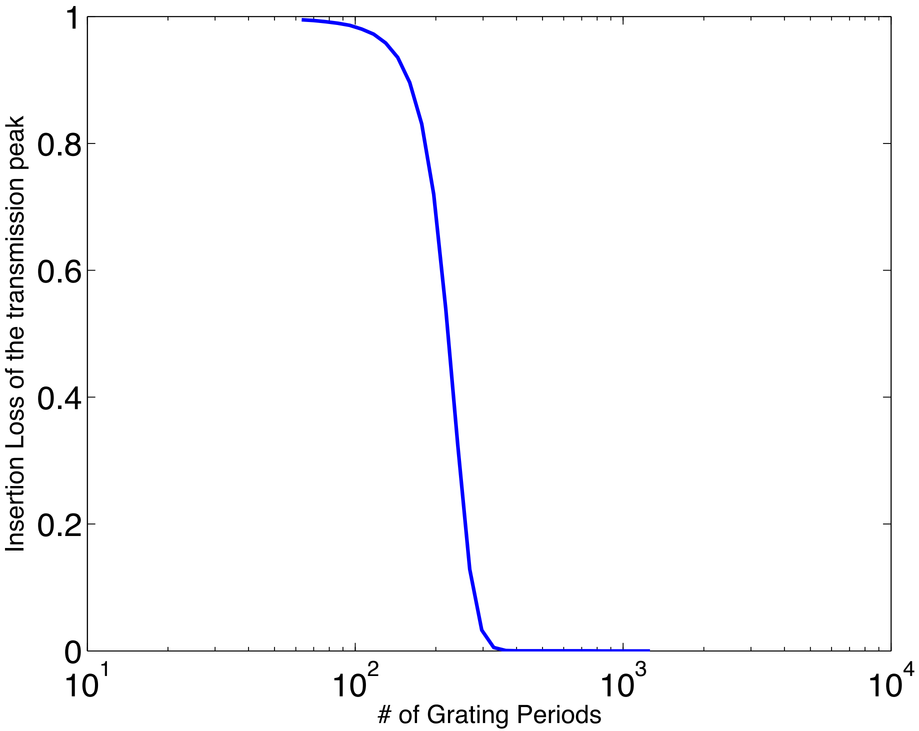
Transmission vs. N



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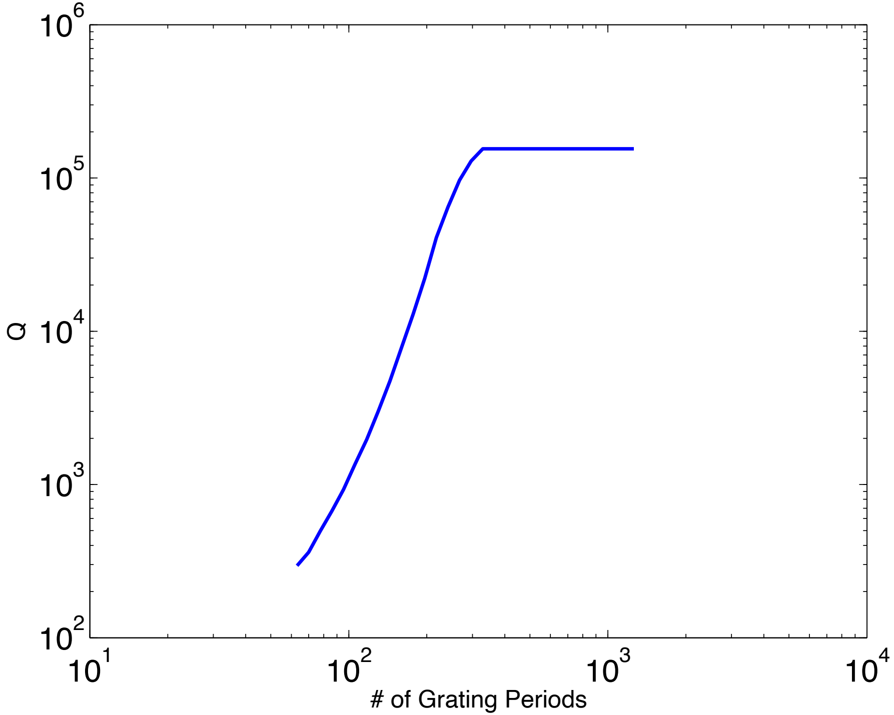
Q vs. N



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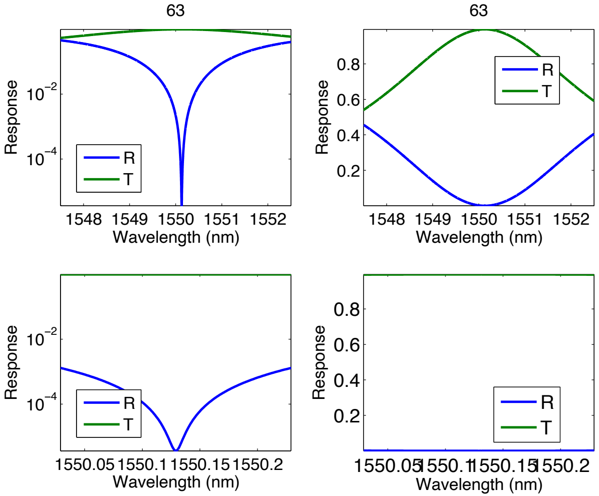
Zoomed in:



Log scale **Linear scale**

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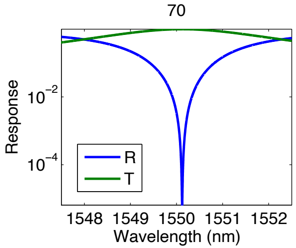
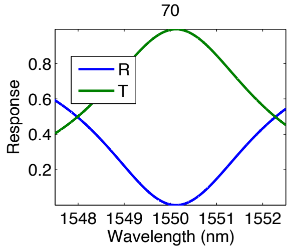
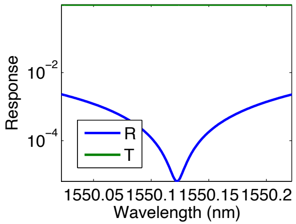
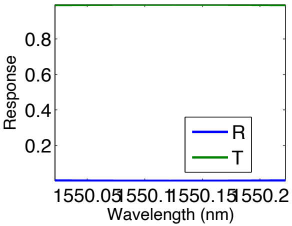




Zoomed in:





Log scale **Linear scale**

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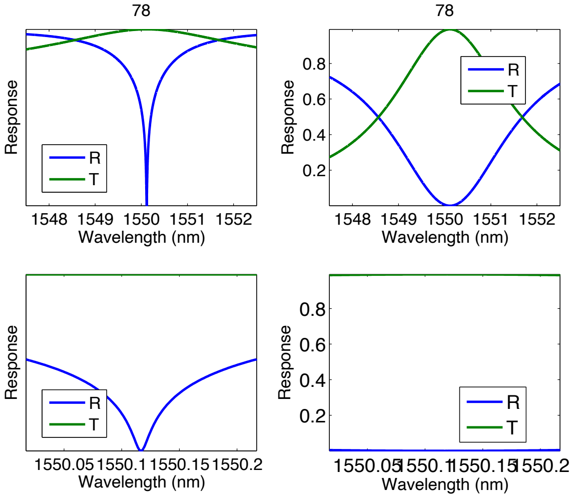

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Log scale **Linear scale**

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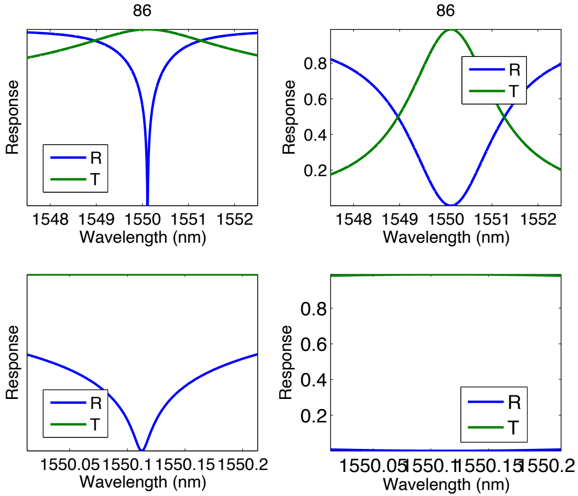

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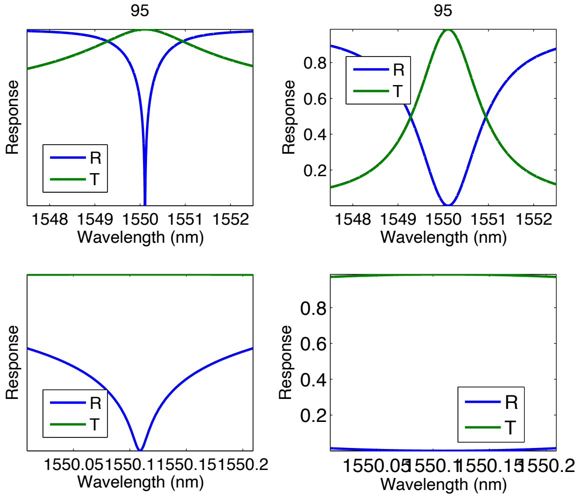

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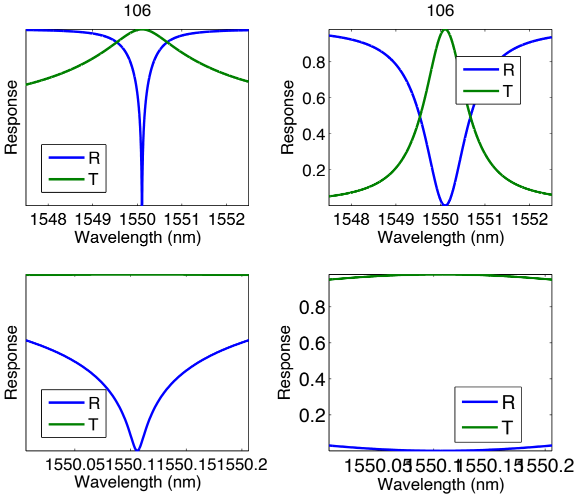

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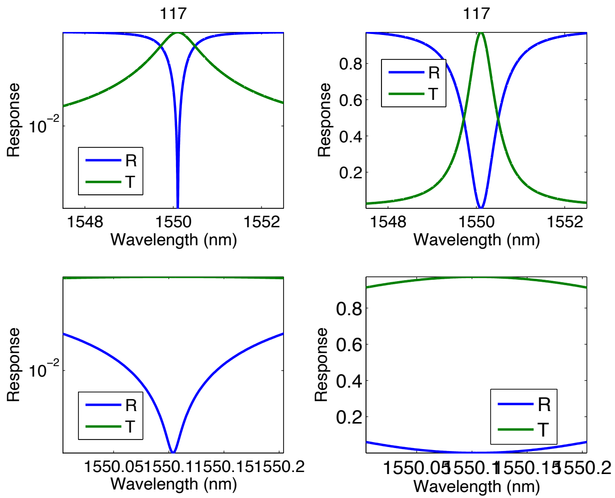

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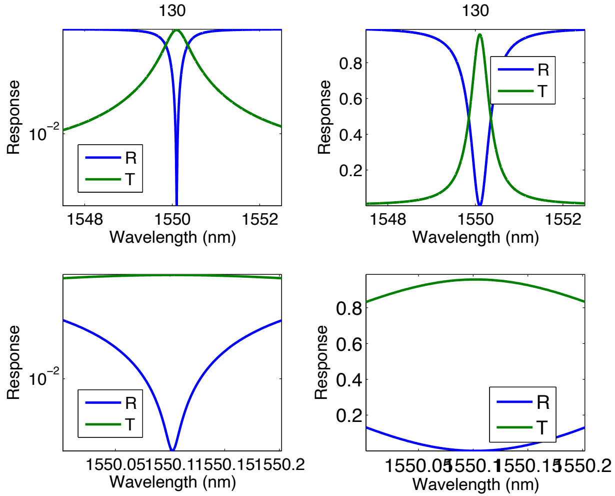

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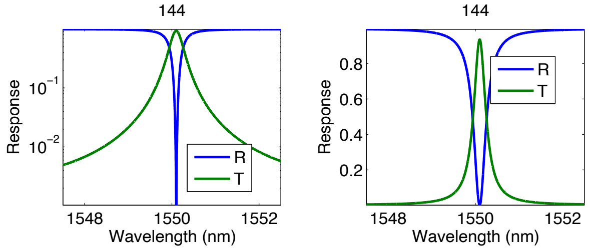

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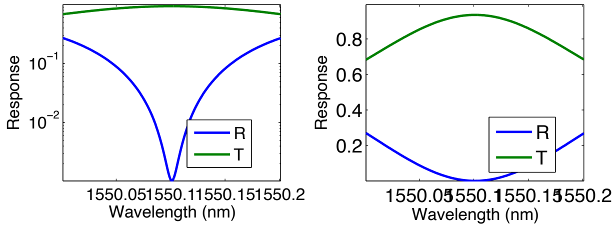

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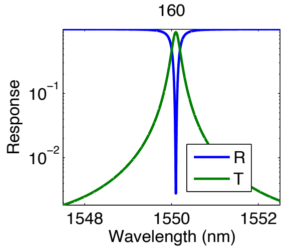

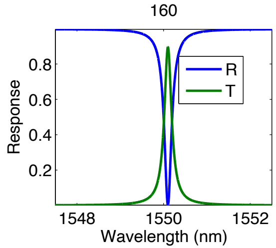

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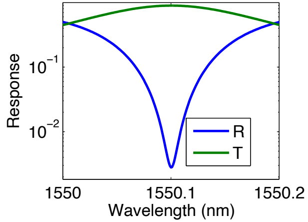

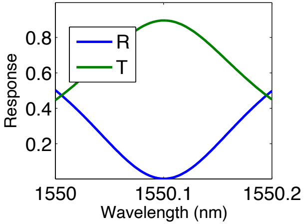

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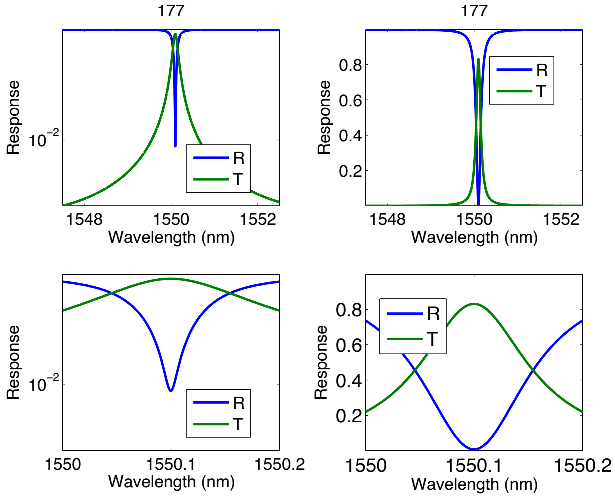

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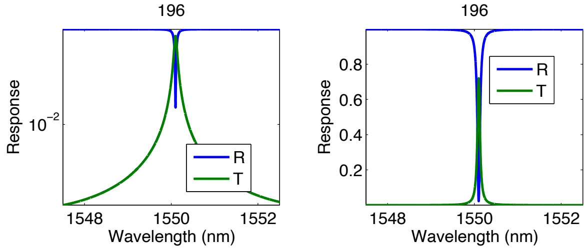

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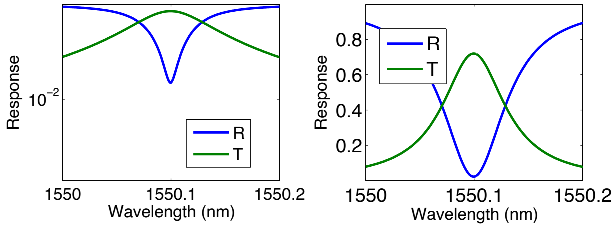

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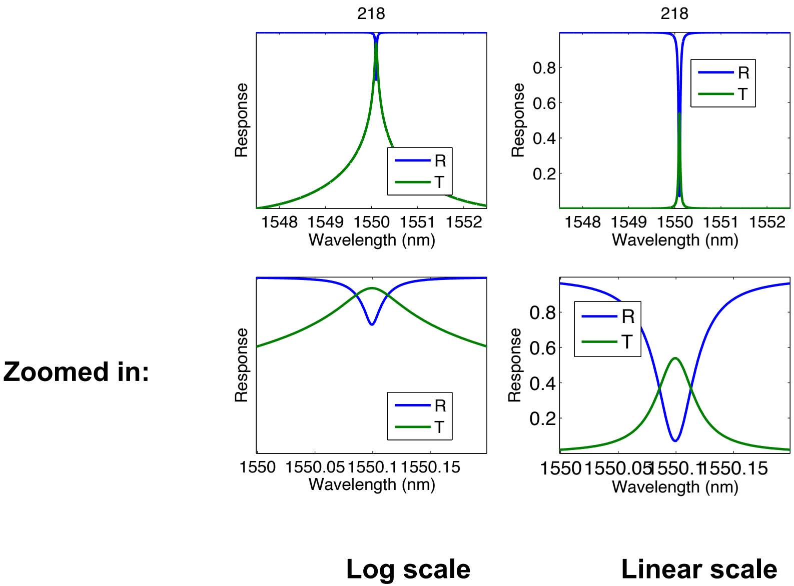
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Wavelength (nm)
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Wavelength (nm)
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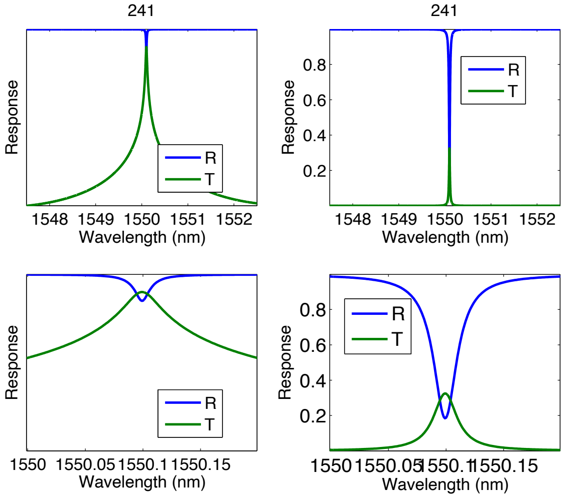

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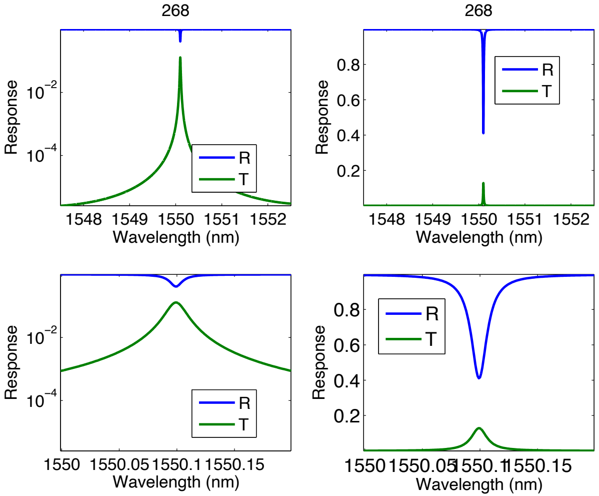

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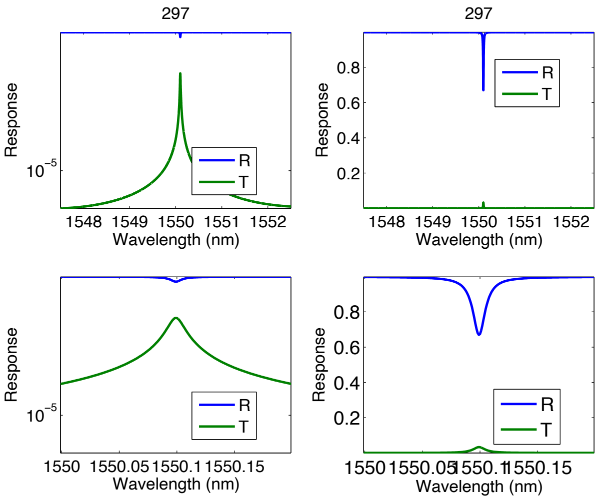

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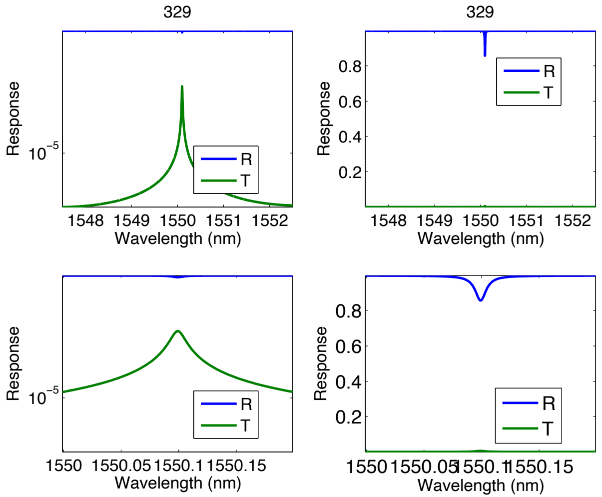

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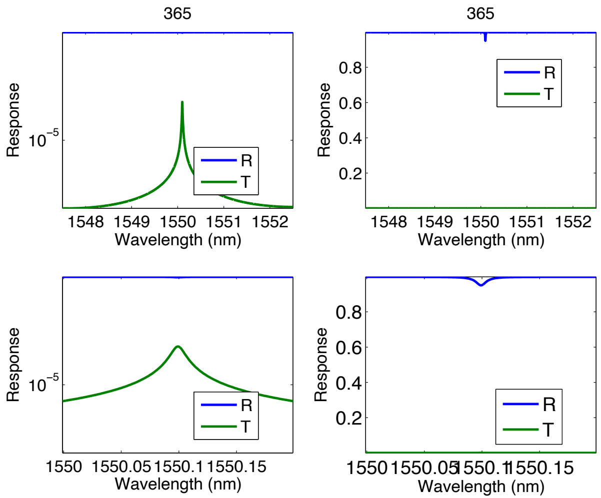

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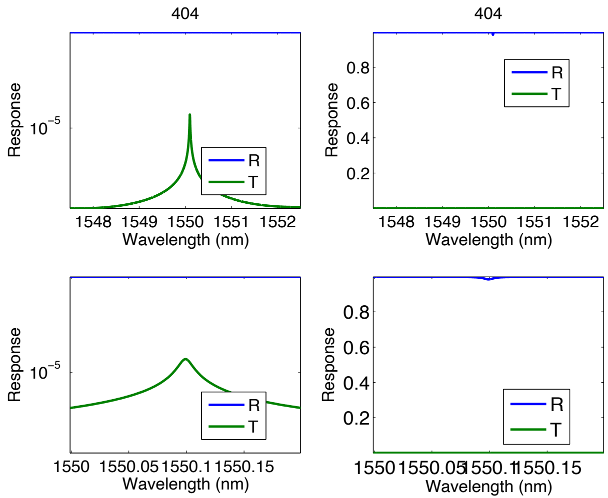
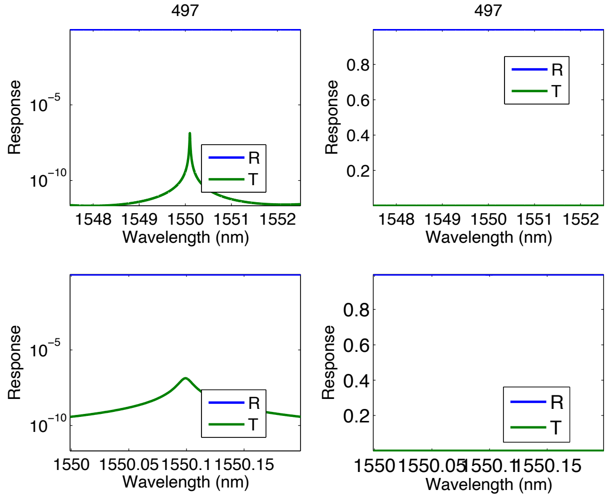


Log scale **Linear scale**



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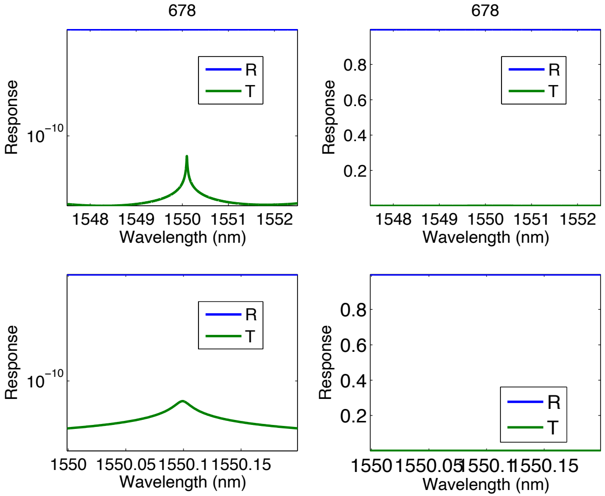

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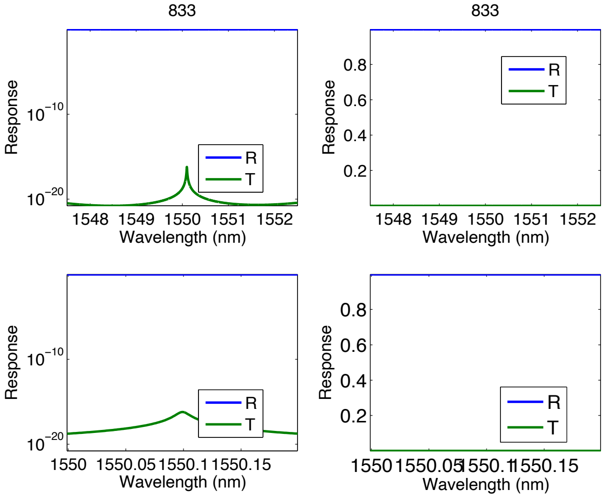

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Design considerations

- Uniform Bragg grating:
  - Δn (Δw), impact on
    - bandwidth of Bragg grating mirror
    - reflectivity
  - L, impact on
    - reflectivity
- Cavity using two Bragg Gratings
  - approaches: phase shifted (VCSEL) vs. long cavity
  - trade-off between Q and insertion loss.
    - due to two loss mechanisms – mirror transmission and internal propagation loss.
- Project report – include models for:
  - simple FP; Uniform Bragg; Bragg cavity.

Project Discussions

- MATLAB vs. INTERCONNECT – matching or not?
  - Missing FP ripples from Grating couplers
  - Shift in wavelength - different waveguide models
  - Quality factor, BW…
- How close to 1310 nm do we have to be?
  - aim for 1290 to 1330 nm
  - Bracket your designs: create a parameter sweep
- How will the cavity length affects the spectrum / shifts?
- How to obtain a high Q factor in a design? Using formula?
  - bracket, aim for Q = 20,000 to 150,000. Transmission 0.01 to 0.9 (excess insertion loss of -20 dB to 1 dB)
- Simulating multiple designs
- What can go wrong in the fabrication / design?
- What parameters / different designs to make in the space given?
  - Identical design – what happens during fabrication? +/- lambda_bragg 20 nm
  - [bracket] — what parameters are expected to vary? 1) neff, 2) Δn, 3) loss.
- Design review / peer assessment